Math 181 Honors Exam 2 Version A

1. Define what it means for a bounded function f on $[a, b]$ to be integrable.

2. Define $\lim_{x \to p} f(x) = L$ in terms of δ and ϵ.

3. Show that $f(x) = x^2$ is continuous at $p = 2$ using the δ-ϵ definition of continuity.

4. Define the derivative $f(x)$ in terms of limits.

5. Let f be differentiable at the point p. Show that f is continuous at p.

6. Let f and g be differentiable on $\mathbf{R}$ and define $w(x) = f(x)g(x)$. Use limit laws to show that $w'(x) = f'(x)g(x) + f(x)g'(x)$.

7. Find the values of the following integrals. For full credit show your work.

(i) $\displaystyle\int_0^2 8x^3\,dx$

(ii) $\displaystyle\int_0^{\pi/12} \sin(3x)\,dx$

(iii) $\displaystyle\int_{-2}^{2} \big|(x-1)(x+2)\big|\,dx$

8. Find the following limits. For full credit show your work.

(i) $\lim\limits_{x \to 0} \dfrac{5x^2 - x + 3}{x + 2}$

(ii) $\lim\limits_{x \to 1} \dfrac{\sin(x^2 - 1)}{x - 1}$

(iii) $\lim\limits_{h \to 0} \left(\dfrac{1}{h^2 + 5h} - \dfrac{1}{5h} \right)$

9. Find the following derivatives. For full credit show your work.

(i) $\dfrac{d}{dx}\left(x^4 + 5x^3 - 17x^2 + x - 13\right)$

(ii) $\dfrac{d}{dx}\left(\sqrt{x} \cdot \sin x\right)$

(iii) $\dfrac{d}{dx}\left(\dfrac{1}{\cos x}\right)$

(iv) $\dfrac{d}{dx} \sin\left(3x + 2\cos(1 + 5x)\right)$

10. Extra Credit Problems

(i) Let f and g be differentiable on $\mathbf{R}$ and define $w(x) = (f \circ g)(x)$. Use limit laws to show that $w'(x) = f'(g(x))g'(x)$.

(ii) Let f be continuous on $\mathbf{R}$ and define $A(x) = \int_0^x f(t)\,dt$. Show $A'(x) = f(x)$ using the definition of derivative.