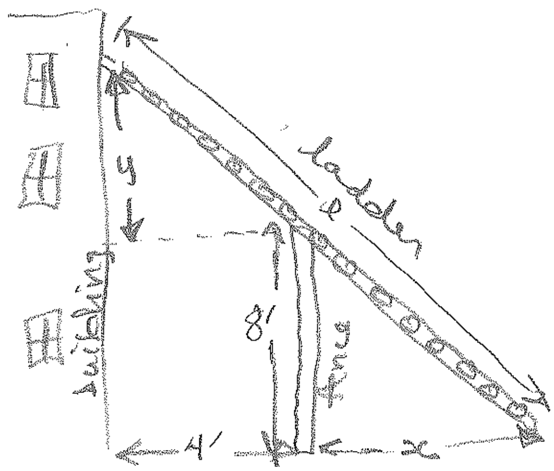
22. A fence 8 ft tall runs parallel to a tall building at a distance of 4 ft from the building. What is the length of the shortest ladder that will reach from the ground over the fence to the wall of the building?



similar triangles:

$$\frac{y}{8} = \frac{4}{x}$$

length of ladder.

$$\ell^2 = (4+x)^2 + (8+y)^2 = (4+x)^2 + 64\left(1+\frac{4}{x}\right)^2$$

Minimizing ℓ is the same as minimizing ℓ^2 and ~~then taking the square root~~, Thus

$$\frac{d\ell^2}{dx} = 2(4+x) + 2\cdot 64\left(1+\frac{4}{x}\right)\left(\frac{-4}{x^2}\right) = 0$$

implies

$$x^3(4+x) - 256(x+4) = 0$$

so

$$(4+x)(x^3-256) = 0$$

Thus $x = -4$ or $x = \sqrt[3]{256} = 16$

22. continues

Since $x \geq 0$ then $x = 16$ is where the minimum occurs.

$$l^2 = (4+16)^2 + 64\left(1+\frac{4}{16}\right)^2 = 500$$

Thus $l = \sqrt{500} = 10\sqrt{5}$.