Math 181 Honors Practice Exam 2 Version A

1. Compute the following derivatives using any method.

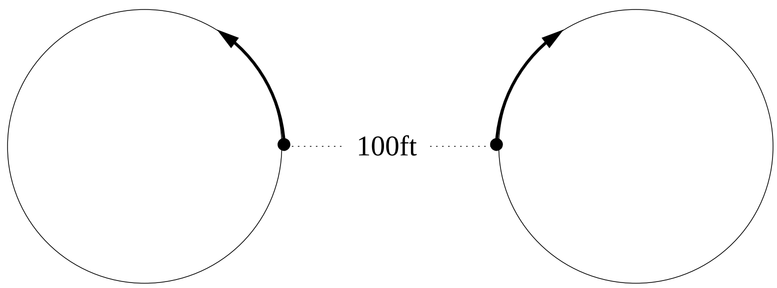
(i) $\dfrac{d}{dx}\left(\dfrac{\sin x}{2+\cos x}\right)$

(ii) $\dfrac{d}{dx} \arcsin\left(\dfrac{1}{\sqrt{1+x^2}}\right)$

(iii) $\dfrac{d}{dx} \ln(1+2x)$

(iv) $\dfrac{d}{dx}\left(x^6+x^5+x^4+x^3+x^2+x+1\right)$

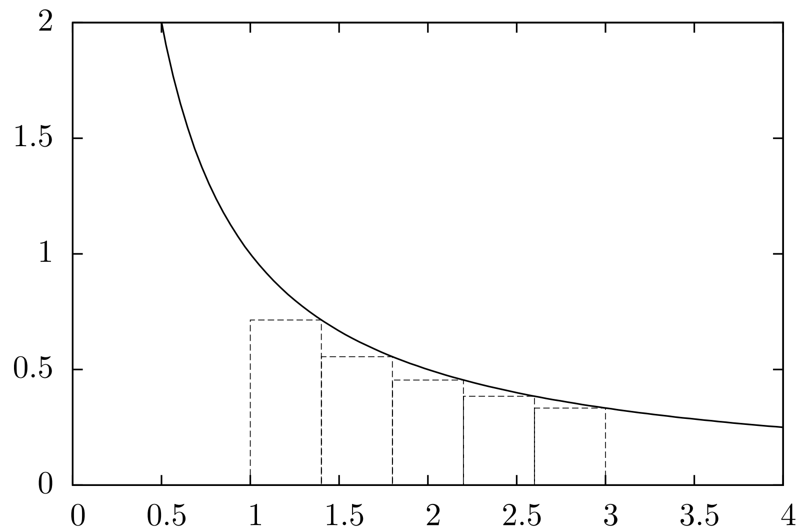
2. Two runners are running on circular tracks which have a circumference of 1320 feet and are 100 feet apart. The runners start at positions opposite and closest to each other as indicated. One runner runs clockwise at a constant rate of 880 feet/minute while the other runs counter clockwise at the same rate. How fast is the distance between the runners changing when each has run 165 feet?



3. A fence 8 feet tall runs parallel to a tall building at a distance of 4 feet from the building. What is the length of the shortest ladder that will reach from the ground over the fence to the wall of the building.

4. Convert the repeating decimal $1.\overline{36}$ to a fraction.

5. Write the sum for the area of the five rectanges shown below that approximate $\ln 3$. Do not add up the terms or attempt to simplify the sum.



6. Solve the inequality $1 < \dfrac{2}{x} - \dfrac{2}{x+1}$.

7. Use induction to show $1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + \cdots + n(n+1) = n(n+1)(n+2)/3$.

8. Solve the following antidifferentiation problems:

(i) Find y so that $\dfrac{dy}{dx} = x^3 + 5$.

(ii) Find w so that $\dfrac{dw}{dt} = \sin t$.

9. Use the δ-ϵ definition of limit to verify that $\lim_{x \to 2} x^3 = 8$.

10. Use the method of increments to find $\dfrac{dy}{dx}$ when $y = \dfrac{1}{x}$.

11. Use implicit differentiation to find $\dfrac{dy}{dx}$ where $y^3 + x^2 = \cos(xy)$.

12. Compute the following limits in any way:

(i) $\lim\limits_{x\to\infty} \dfrac{x-17}{1+x^2}$.

(ii) $\lim\limits_{x\to 0} \dfrac{1-\cos 3x}{x^2}$.

13. Show that

$$\frac{d\arctan x}{dx} = \frac{1}{1+x^2}$$

using the identity $\sec^2 x = 1+\tan^2 x$ and the fact that $\dfrac{d\tan x}{dx} = \sec^2 x$.