Math 181 Honors Exam 2 Version A

1. Convert $7.\overline{5}$ into a fraction.

2. Solve the equation $\ln(e^{x+1}) = 3$

(A) 1

(B) 2

(C) 3

(D) 4

(E) none of these.

3. Compute $\int_0^{\pi/12} \cos(3x)\, dx$

4. Use induction to show that $\sum_{k=1}^{n} k = n(n+1)/2$.

5. Compute the sum $\sum_{k=5}^{29} (k+1)$

6. State the δ-ϵ definition of $\lim_{x \to p} f(x) = A$.

7. Show that $f(x) = 1/x$ is continuous at the point $p = 1$ using the δ-ϵ definition of continuity.

8. Compute the following limits:

(i) $\lim_{x \to 2} (x^2 - 7)$.

(ii) $\lim_{x \to 4} \frac{\sin(x-4)}{\sqrt{x} - 2}$.

(iii) $\lim_{h \to 0} \left(\frac{1}{h^2 + 4h} - \frac{1}{4h} \right)$.

9. Explain the method of increments.

10. Suppose $y = uv$ hwere u and v depend on x Show that

$$\frac{dy}{dx} = \frac{du}{dx} v + u \frac{dv}{dx}$$

using the method of increments.

11. Compute the following derivatives:

(i) $\dfrac{d}{dx}(x^5 + 3x^4 - x^2 + 8)$

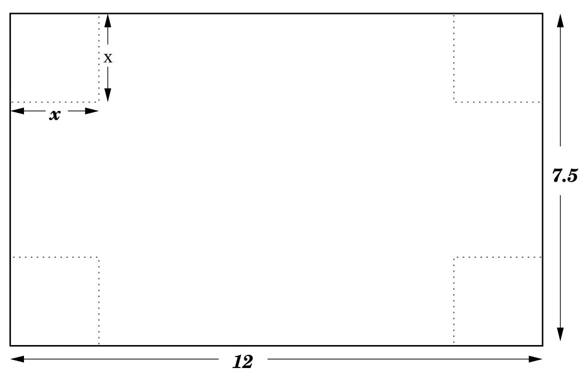
(ii) $\dfrac{d}{dx}\sin(\ln x)$

(iii) $\dfrac{d}{dx}\displaystyle\int_1^{1+x^2} \frac{\sin t}{t}\,dt$

12. Using the method of increments to prove the Fundamental Theorem of Calculus:

> **Theorem 5.1** Let f be a function that is integrable on $[a, x]$ for each x in $[a, b]$. Let c be such that $a \le c \le b$ and define a new function A as follows:
>
> $$A(x) = \int_c^x f(t)dt \qquad \text{if} \qquad a \le x \le b.$$
>
> Then the derivative $A'(x)$ exists at each point in the open interval (a, b) where f is continuous, and for such x we have $A'(x) = f(x)$.

13. An open box is made from a rectangular piece of material by removing equal squares at each corner and turning up the sides. Find the dimensions of the box of largest volume that can be made in this manner if the material has sides 7.5 and 12.



14. Convert the repeating decimal $1.\overline{27}$ to a fraction.

15. Use the δ-ϵ definition of limit to verify that $\lim\limits_{x\to 4}\sqrt{x} = 2$.

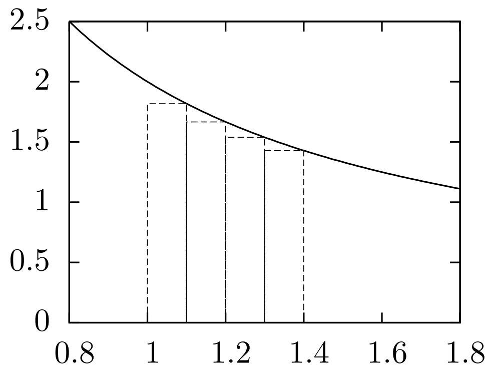
16. Suppose $\lim\limits_{x\to 2} f(x) = 5$. Use the δ-ϵ definition of limit to verity $\lim\limits_{x\to 2} xf(x) = 10$.

17. Use the definition method of increments to find dy/dx where $y = 1/x$.

18. Use the limit $\lim\limits_{h\to 0}\dfrac{\sin h}{h} = 1$ to compute $\lim\limits_{h\to 0}\dfrac{1-\cos 3h}{h^2}$.

19. Use the fundamental theorem of calculus to compute $\frac{d}{dt}\int_0^t \sin(x^2)\,dx$.

20. Write the sum for area of the four rectangles shown below that approximate the area under the curve $f(x) = 2/x$ between $x = 1$ and $x = 7/5$. Do not add up the terms or attempt to simplify the sum.



21. Find the following derivatives.

(i) $\frac{d}{dx}\arctan\left(2\sqrt{x}+3\right)$

(ii) $\frac{d}{dx}\sin(x^2+3)$

(iii) $\frac{d}{dx}\tan\left(\frac{x}{x^4+7}\right)$

(iv) $\frac{d^4}{dx^4}(x^4+17x^3+27x^2+13x+7)$

22. Find the following sums.

(i) $\sum_{k=3}^{9} k$

(ii) $\sum_{k=n^2}^{n^3+1} (2k+5)$

(iii) $\sum_{k=n}^{n+17} k^2$

(iv) $\sum_{k=1}^{31}(k-16)^2$

23. Find the following definite and indefinite integrals.

(i) $\displaystyle\int (x^2 - 1)^2 \, dx$

(ii) $\displaystyle\int x \cos(2x^2 + 5) \, dx$

(iii) $\displaystyle\int_0^1 5x^{99} \, dx$

(iv) $\displaystyle\int_0^2 x\sqrt{x+3} \, dx$

24. A certain poster requires 96 in^2 for the printed message and must have 3-in margins at the top and bottom and a 2-in margin on each side. Find the overall dimensions of the poster if the amount of paper used is a minimum.

25. A woman raises a bucket of cement to a platform 40 ft above her head by means of a rope 80 ft long that passes over a pulley on the platform. If she holds her end of the rope firmly at head level and walks away at 5 ft/s, how fast is the bucket rising when she is 30 ft away from the spot directly below the pulley?