Math 181 Honors Quiz 3 Version A

Dear class,

Here are my solutions to the first page of Quiz 3.

Sincerely, Eric

**1.** The binomial theorem states that

$$(x_0 + \Delta x)^n = \sum_{k=0}^{n} \binom{n}{k} x_0^{n-k} (\Delta x)^k \qquad \text{where} \qquad \binom{n}{k} = \frac{n!}{k!(n-k)!}.$$

Use the binomial theorem and the method of increments to show that

$$\frac{dy}{dx} = nx^{n-1} \qquad \text{for} \qquad y = x^n.$$

**Solution:** By the binomial theorem

$$\begin{aligned} y_0 + \Delta y = (x_0 + \Delta x)^n &= \sum_{k=0}^{n} \binom{n}{k} x_0^{n-k} (\Delta x)^k \\ &= x_0^n + nx_0^{n-1}\Delta x + \sum_{k=2}^{n} \binom{n}{k} x_0^{n-k} (\Delta x)^k. \end{aligned}$$

Subtracting $y_0 = x_0^n$, dividing by $\Delta x$ and taking limits yields

$$\frac{\Delta y}{\Delta x} = nx_0^{n-1} + \sum_{k=2}^{n} \binom{n}{k} x_0^{n-k} (\Delta x)^{k-1} \to nx_0^{n-1} \qquad \text{as} \qquad \Delta x \to 0.$$

Therefore

$$\frac{dy}{dx} = nx^{n-1}.$$

**2.** State the definition of

$$\lim_{x \to a} f(x) = L$$

in terms of $\delta$ and $\epsilon$.

**Solution:** For every $\epsilon > 0$ there exists $\delta > 0$ such that $0 < |x - a| < \delta$ implies $|f(x) - L| < \epsilon$.