Math 181 Honors Quiz 4 Version A

— Key —

1. Solve the inequality $\dfrac{2}{x+2} > \dfrac{x+2}{2}$.

$$\frac{x+2}{2} - \frac{2}{x+2} < 0$$

$$\frac{(x+2)^2 - 4}{2(x+2)} < 0$$

$$\frac{x^2+4x}{2(x+2)} < 0$$

$$\frac{x(x+4)}{2(x+2)} < 0$$

		-4		-2		0	
(x)	$-$		$-$		$-$		$+$
$x+4$	$-$		$+$		$+$		$+$
$x+2$	$-$		$-$		$+$		$+$
	$-$		$+$		$-$		$+$

Solution:

$x \in (-\infty, -4) \cup (-2, 0)$

2. Use the method of increments to compute $\dfrac{dy}{dx}$ where $y = \sqrt{x}$.

$$y_0 + \Delta y = \sqrt{x_0 + \Delta x}$$

$$(y_0 + \Delta y)^2 = x_0 + \Delta x$$

$$y_0^2 + 2y_0\Delta y + (\Delta y)^2 = x_0 + \Delta x$$

$$y_0^2 = x_0$$

$$2y_0\Delta y + (\Delta y)^2 = \Delta x$$

$$\Delta y(2y_0 + \Delta y) = \Delta x$$

$$\frac{\Delta y}{\Delta x} = \frac{1}{2y_0 + \Delta y} \to \frac{1}{2y_0} \text{ as } \Delta x \to 0$$

Thus $\dfrac{dy}{dx} = \dfrac{1}{2y} = \dfrac{1}{2\sqrt{x}}$

3. Solve the inequality $x^2 - 5x + 2 > 0$.

$$x^2 - 5x + 2 = 0$$

$a = 1 \quad b = -5 \quad c = 2$

$$x = \frac{5 \pm \sqrt{25 - 8}}{2} = \frac{5 \pm \sqrt{17}}{2}$$

$$\left(x - \frac{5+\sqrt{17}}{2}\right)\left(x - \frac{5-\sqrt{17}}{2}\right) > 0$$

	$\left(-\infty, \frac{5-\sqrt{17}}{2}\right)$	$\left(\frac{5-\sqrt{17}}{2}, \frac{5+\sqrt{17}}{2}\right)$	$\left(\frac{5+\sqrt{17}}{2}, \infty\right)$
$x - \frac{5+\sqrt{17}}{2}$	$-$	$-$	$+$
$x - \frac{5-\sqrt{17}}{2}$	$-$	$+$	$+$
	$+$	$-$	$+$

Solution:

$$x \in \left(-\infty, \frac{5-\sqrt{17}}{2}\right) \cup \left(\frac{5+\sqrt{17}}{2}, \infty\right)$$

4. Use the δ-ϵ definition of limit to verify $\lim_{x \to 2} x^2 = 4$.

Let $\varepsilon > 0$ be arbitrary and choose $\delta = \min(1, \varepsilon/5)$

Then $0 < |x-2| < \delta$ implies

$-1 < x - 2 < 1$ so $3 < x + 2 < 5$

and therefore

$$|x^2 - 4| = |x-2||x+2| < \delta |x+2|$$

$$= \delta(x+2) < 5\delta \leq \varepsilon.$$