Ellis & Gulick, Calculus with Analytic Geometry 2nd Ed.

EXERCISES 3.8

1. Suppose the radius of a spherical balloon is shrinking at $\frac{1}{2}$ inch per minute. How fast is the volume decreasing when the radius is 4 inches?
2. Suppose a snowball remains spherical while it melts, with the radius shrinking at one inch per hour. How fast is the volume of the snowball decreasing when the radius is 2 inches?
3. Suppose the volume of the snowball in Exercise 2 shrinks at the rate of $dV/dt = -2/V$ (cubic inches per hour). How fast is the radius changing when the radius is $\frac{1}{2}$ inch?

4. A spherical balloon is inflated at the rate of 3 cubic inches per minute. How fast is the radius of the balloon increasing when the radius is 6 inches?

5. Suppose a spherical balloon grows in such a way that after t seconds, $V = 4\sqrt{t}$ (cubic inches). How fast is the radius changing after 64 seconds?

6. A spherical balloon is losing air at the rate of 2 cubic inches per minute. How fast is the radius of the balloon shrinking when the radius is 8 inches?

7. Water leaking onto a floor creates a circular pool whose area increases at the rate of 3 square inches per minute. How fast is the radius of the pool increasing when the radius is 10 inches?

8. A point moves around the circle $x^2 + y^2 = 9$. When the point is at $(-\sqrt{3}, \sqrt{6})$, its x coordinate is increasing at the rate of 20 units per second. How fast is its y coordinate changing at that instant?

9. A ladder 15 feet long leans against a vertical wall. If the bottom end of the ladder is pulled away from the wall at a rate of 1 foot per second, at what rate does the top of the ladder slip down the wall when the bottom of the ladder is 5 feet from the wall?

10. Suppose the top of the ladder in Exercise 9 is being pushed up the wall at the rate of 1 foot per second. How fast is the base of the ladder approaching the wall when it is 3 feet from the wall?

11. A board 5 feet long slides down a wall. At the instant the bottom end is 4 feet from the wall, the other end is moving down the wall at the rate of 2 feet per second. At that moment,
    a. how fast is the bottom end sliding along the ground?
    b. how fast is the area of the region between the board, ground, and wall changing?

12. Suppose the water in Example 4 is poured in at the rate of $\frac{3}{2}$ cubic inches per second. How fast is the water level rising when the water is 2 inches deep?

13. Suppose that the water level in Example 4 is rising at $\frac{1}{2}$ inch per second. How fast is the water being poured in when the water has a depth of 2 inches?

14. Water is released from a conical tank with height 50 feet and radius 30 feet and falls into a rectangular tank whose base has an area of 400 square feet (Figure 3.29). The rate of release is controlled so that when the height of the water in the conical tank is x feet, the height is decreasing at the rate of $50 - x$ feet per minute. How fast is the water level in the rectangular tank rising when the height of the water in the conical tank is 10 feet? (*Hint:* The total amount of water in the two tanks is constant.)

15. A water trough is 12 feet long, and its cross section is an equilateral triangle with sides 2 feet long. Water is pumped into the trough at a rate of 3 cubic feet per minute. How fast is the water level rising when the depth of the water is $\frac{1}{2}$ foot?

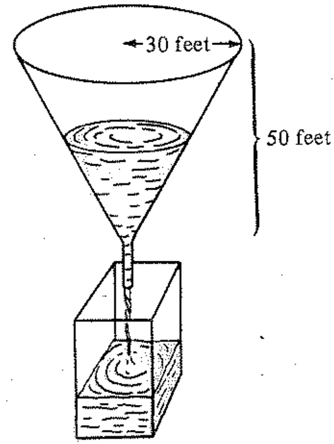

FIGURE 3.29

16. A rope is attached to the bow of a sailboat coming in for the evening. Assume that the rope is drawn in over a pulley 5 feet higher than the bow at the rate of 2 feet per second, as shown in Figure 3.30. How fast is the boat docking when the length of rope from bow to pulley is 13 feet?

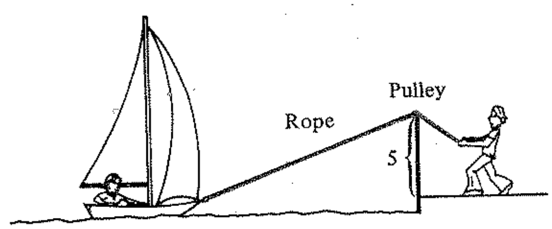

FIGURE 3.30

17. Suppose the rope in Exercise 16 is pulled so that the boat docks at a constant rate of 2 feet per second. How fast is the rope being pulled in when the boat is 12 feet from the dock?

18. As in Exercise 16, assume that the boat is pulled in by a rope attached to the bow passing through a pulley 5 feet above the bow. Assume also that the distance between the bow and the dock decreases as the cube root of the distance; that is, if the distance at time t is y feet, then $dy/dt = -y^{1/3}$ (feet per second). How fast is the length of the rope shrinking when the bow is 8 feet from the dock?

19. A spotlight is on the ground 100 feet from a building that has vertical sides. A person 6 feet tall starts at the spotlight and walks directly toward the building at a rate of 5 feet per second.
   a. How fast is the top of the person's shadow moving down the building when the person is 50 feet away from it?
   b. How fast is the top of the shadow moving when the person is 25 feet away?

20. A kite 100 feet above the ground is being blown away from the person holding its string, in a direction parallel to the ground and at the rate of 10 feet per second. At what rate must the string be let out when the length of string already let out is 200 feet?

21. When a rocket is two miles high, it is moving vertically upward at a speed of 300 miles per hour. At that instant, how fast is the angle of elevation of the rocket increasing, as seen by an observer on the ground 5 miles from the launching pad?

22. A street light 16 feet high casts a shadow on the ground from a ball that is dropped from a height of 16 feet but 15 feet from the light. How fast is the shadow moving along the ground when the ball is 5 feet from the ground. (*Note:* The distance s from the ball to the ground t seconds after release is given by the equation $s = 16 - 16t^2$.)

23. A person is pushing a box up the ramp in Figure 3.31 at the rate of 3 feet per second. How fast is the box rising?

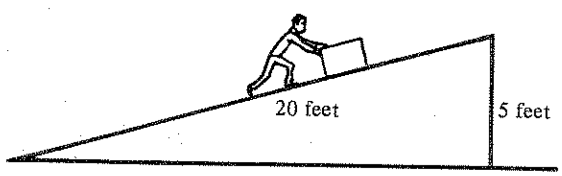

FIGURE 3.31

24. Maple and Main Streets are straight and perpendicular to each other. A stationary police car is located on Main Street $\frac{1}{4}$ mile from the intersection of the two streets. A sports car on Maple Street approaches the intersection at the rate of 40 miles per hour. How fast is the distance between the two cars decreasing when the moving car is $\frac{1}{3}$ mile from the intersection?

25. Suppose in Exercise 24 that as the sports car approaches the intersection, the distance between the sports car and the police car decreases at 25 miles per hour. How far from the intersection would the sports car be at the moment when it is traveling 40 miles per hour?

26. A helicopter flies parallel to the ground at an altitude of $\frac{1}{2}$ mile and at a speed of 2 miles per minute. If the helicopter flies along a straight line that passes directly over the White House, at what rate is the distance between the helicopter and the White House changing 1 minute after the helicopter flies over the White House?

27. A Flying Tiger is making a nose dive along a parabolic path having the equation $y = x^2 + 1$, where x and y are measured in feet. Assume that the sun is directly above the y axis, that the ground is the x axis, and that the distance from the plane to the ground is decreasing at the constant rate of 100 feet per second. How fast is the shadow of the plane moving along the ground when the plane is 2501 feet above the earth's surface? Assume that the sun's rays are vertical.

28. Boyle's Law states that if the temperature of a gas remains constant, then the pressure p and the volume V of the gas satisfy the equation $pV = c$, where c is a constant. If the volume is decreasing at the rate of 10 cubic inches per second, how fast is the pressure increasing when the pressure is 100 pounds per square inch and the volume is 20 cubic inches?

29. The tortoise and the hare are having their fabled footrace, each moving along a straight line. The tortoise, moving at a constant rate of 10 feet per minute, is 4 feet from the finish line when the hare wakes up 5001 feet from the finish line and darts off after the tortoise. Let x be the distance from the tortoise to the finish line, and suppose

$$y = 5001 - 2500\sqrt{4 - x}$$

is the distance from the hare to the finish line.
   a. How fast is the hare moving when the tortoise is 3 feet from the finish line?
   b. Who wins? By how many feet?

30. A 10-foot square sign of negligible thickness revolves about a vertical axis through its center at a rate of 10 revolutions per minute. An observer far away sees it as a rectangle of variable width. How fast is the width changing when the sign appears to be 6 feet wide and is increasing in width? (*Hint:* View the sign from above, and consider the angle it makes with a line pointing toward the observer.)

31. Suppose a deer is standing 20 feet from a highway on which a car is traveling at a constant rate of v feet per second. Let θ be the angle made by the highway and the line of sight from a passenger to the deer (Figure 3.32).

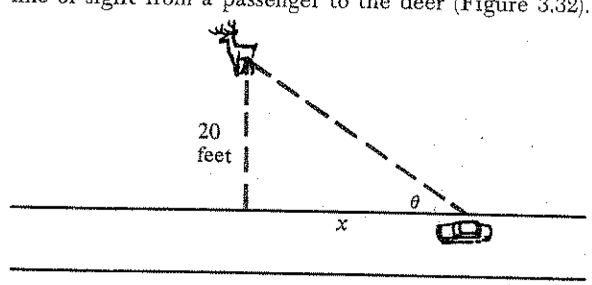

FIGURE 3.32

Show that

$$\frac{d\theta}{dt} = \frac{20v}{400 + x^2}$$

(Notice that for x close to 0, $d\theta/dt$ is approximately $v/20$, and thus for the passenger to keep the deer in focus, the passenger's eyes must rotate at the approximate rate of $v/20$ feet per second. This suggests why at large velocities it may be impossible to keep a stationary object near the highway in focus.)

°32. At night a patrol boat approaches a point on shore along the curve $y = -\frac{1}{2}x^3$, as indicated in Figure 3.33. If the boat moves along the curve so that $dx/dt = -x$, and if its spotlight is pointed straight ahead, determine how fast the illuminated spot on the shore moves when $x = -2$. (*Hint:* You will need to find the x intercept of the line tangent to the curve $y = -\frac{1}{2}x^3$.)

°33. A deer 5 feet long and 6 feet tall, whose rump is 4 feet above ground as in Figure 3.34, approaches a street light with lamp 20 feet above ground. If the deer proceeds at 3 feet per second, how fast is its shadow changing when the front of the deer is

a. 48 feet from the street light?

b. 24 feet from the street light?

(*Hint:* In parts (a) and (b), determine which yields the shadow, the head or the rump of the deer.)

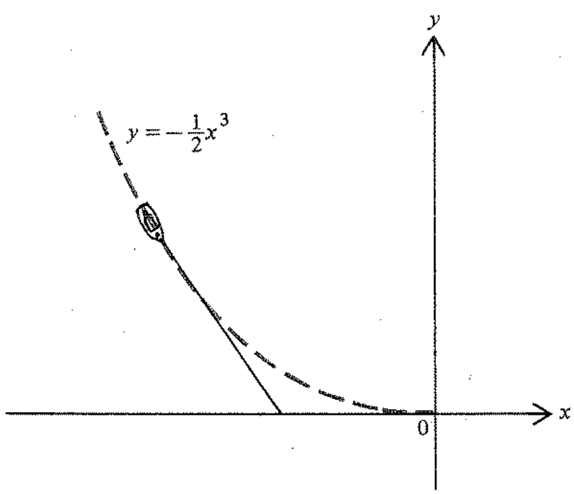

FIGURE 3.33

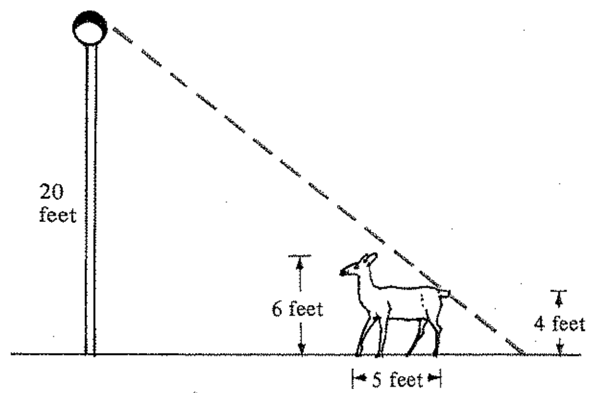

FIGURE 3.34