Simmons, Calculus with analytic Geometry 2nd Ed

PROBLEMS

1 A stone dropped into a pond sends out a series of concentric ripples. If the radius r of the outer ripple increases steadily at the rate of 6 ft/s, find the rate at which the area of disturbed water is increasing (a) when $r = 10$ ft, and (b) when $r = 20$ ft.

2 A large spherical snowball is melting at the rate of 2π ft^3/h. At the moment when it is 30 inches in diameter, determine (a) how fast the radius is changing, and (b) how fast the surface area is changing.

3 Sand is being poured onto a conical pile at the constant rate of 50 ft^3/min. Frictional forces in the sand are such that the height of the pile is always equal to the radius of its base. How fast is the height of the pile increasing when the sand is 5 ft deep?

4 A girl 5 ft tall is running at the rate of 12 ft/s and passes under a street light 20 ft above the ground. Find how rapidly the tip of her shadow is moving when she is (a) 20 ft past the street light, and (b) 50 ft past the street light.

5 In Problem 4, find how rapidly the length of the girl's shadow is increasing at each of the stated moments.

6 A light is at the top of a pole 80 ft high. A ball is dropped from the same height from a point 20 ft away from the light. Find how fast the shadow of the ball is moving along the ground (a) 1 second later; (b) 2 seconds later. (Assume that the ball falls $s = 16t^2$ feet in t seconds.)

7 A woman raises a bucket of cement to a platform 40 ft above her head by means of a rope 80 ft long that passes over a pulley on the platform. If she holds her end of the rope firmly at head level and walks away at 5 ft/s, how fast is the bucket rising when she is 30 ft away from the spot directly below the pulley?

8 A boy is flying a kite at a height of 80 ft, and the wind is blowing the kite horizontally away from the boy at the rate of 20 ft/s. How fast is the boy paying out string when the kite is 100 ft away from him?

9 A boat is being pulled in to a dock by means of a rope with one end tied to the bow of the boat and the other end passing through a ring attached to the dock at a point 5 ft higher than the bow of the boat. If the rope is being pulled in at the rate of 4 ft/s, how fast is the boat moving through the water when 13 ft of rope are out?

10 A trough is 10 ft long and has a cross section in the shape of an equilateral triangle 2 ft on each side. If water is being pumped in at the rate of 20 ft^3/min, how fast is the water level rising when the water is 1 ft deep?

11 A spherical meteorite enters the earth's atmosphere and burns up at a rate proportional to its surface area. Show that its radius decreases at a constant rate.

12 A point moves around the circle $x^2 + y^2 = a^2$ in such a way that the x-component of its velocity is given by $dx/dt = -y$. Find dy/dt and decide whether the direction of the motion is clockwise or counterclockwise.

13 A car moving at 60 mi/h along a straight road passes under a weather balloon rising vertically at 20 mi/h. If the balloon is 1 mi up when the car is directly beneath it, how fast is the distance between the car and the balloon increasing 1 minute later?

14 Most gases obey Boyle's law: If a sample of the gas is held at a constant temperature while being compressed by a piston in a cylinder, then its pressure p and volume V are related by the equation $pV = c$, where c is a constant. Find dp/dt in terms of p and dV/dt.

15 At a certain moment a sample of gas obeying Boyle's law (Problem 14) occupies a volume of 1000 in^3 at a pressure of 10 lb/in^2. If this gas is being compressed isothermally at the rate of 12 in^3/min, find the rate at which the pressure is increasing at the instant when the volume is 600 in^3.

*16 A ladder 20 ft long is leaning against a wall 12 ft high, with its top projecting over the wall. Its bottom is being pulled away from the wall at the constant rate of 5 ft/min. Find how rapidly the top of the ladder is approaching the ground (a) when 5 ft of the ladder projects over the wall; (b) when the top of the ladder reaches the top of the wall.

17 A conical party hat made of cardboard has a radius of 4 in and a height of 12 in. When filled with beer, it leaks at the rate of 4 in^3/min. At what rate is the level of beer falling (a) when the beer is 6 in deep? (b) when the hat is half empty?

18 A hemispherical bowl of radius 8 in is being filled with water at a constant rate. If the water level is rising at the rate of $\frac{1}{3}$ in/s at the instant when the water is 6 in deep, find how fast the water is flowing in

(a) by using the fact that a segment of a sphere has volume

$$V = \pi h^2 \left(a - \frac{h}{3}\right)$$

where a is the radius of the sphere and h is the height of the segment;

(b) by using the fact that if V is the volume of the water at time t, then

$$\frac{dV}{dt} = \pi r^2 \frac{dh}{dt}$$

where r is the radius of the surface and h is the depth.

19 Water is being poured into a hemispherical bowl of radius 3 in at the rate of 1 in^3/s. How fast is the water level rising when the water is 1 in deep?

***20** In Problem 19, suppose that the bowl contains a lead ball 2 inches in diameter, and find how fast the water level is rising when the ball is half submerged.

21 Assume that a snowball melts in such a way that its volume decreases at a rate proportional to its surface area. If half the original snowball has melted away after 2 hours, how much longer will it take for the snowball to disappear completely?

22 A man in a hot air balloon is rising at the rate of 20 ft/s. How fast is the distance to the horizon increasing when the balloon is 2000 ft high? Assume that the earth is a sphere of radius 4000 mi.

23 A drawbridge with two 20-ft spans is being raised at the rate of 2 radians/min (Fig. 4.36). How fast is the distance between the ends of the spans increasing when they are elevated $\pi/4$ radians?

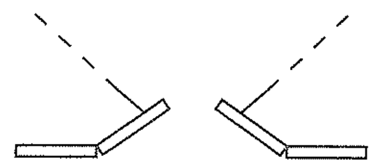

Figure 4.36