Honors Math 181 Homework 7 Version A

1. Let f be a continuous function and $h(x) = f(x-7)$. Use the equality

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{j=1}^{n} f\Big(a + j\frac{b-a}{n}\Big)\frac{b-a}{n}$$

to show that

$$\int_a^b h(x)\,dx = \int_{a-7}^{b-7} f(x)\,dx.$$

2. Use the integral formulas

$$\int_a^b \cos x\,dx = \sin x\Big|_a^b \qquad \int_a^b \sin x\,dx = -\cos x\Big|_a^b \qquad \int_a^b \frac{1}{x}\,dx = \ln x\Big|_a^b$$

$$\int_a^b 1\,dx = x\Big|_a^b \qquad \int_a^b x\,dx = \frac{x^2}{2}\Big|_a^b \qquad \int_a^b x^2\,dx = \frac{x^3}{3}\Big|_a^b$$

and the transformation rules

$$\int_a^b kf(x)\,dx = k\int_a^b f(x)\,dx \qquad \int_a^b \big(f(x)+g(x)\big)\,dx = \int_a^b f(x)\,dx + \int_a^b g(x)\,dx$$

$$\int_a^b f(x/k)\,dx = k\int_{a/k}^{b/k} f(x)\,dx \qquad \int_a^b f(x-k)\,dx = \int_{a-k}^{b-k} f(x)\,dx$$

to find the following integrals:

(i) $\displaystyle\int_{\pi/6}^{\pi/3} \cos x\,dx$

(ii) $\displaystyle\int_{\pi/6}^{\pi/3} \sin 2x\,dx$

(iii) $\displaystyle\int_0^2 \big(1-x^2\big)\,dx$

(iv) $\displaystyle\int_0^2 \big|1-x^2\big|\,dx$

3. Simplify the following sums:

(i) $\displaystyle\sum_{k=1}^{n} \Big(2 + \frac{k}{3n}\Big)$

(ii) $\displaystyle\sum_{k=1}^{n} \big((k+1)^4 - k^4\big)$