Math 181 Quiz 7 Version A

1. Let f be a continuous function on the interval $[a, b]$ where $a < b$. Then

(A) $$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{j=1}^{n} f\left(a + j\frac{b+a}{n}\right)\frac{b+a}{n}.$$

(B) $$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{j=1}^{n} f\left(a + j\frac{b-a}{n}\right)\frac{b-a}{n}.$$

(C) $$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{j=1}^{n} f\left(a - j\frac{b+a}{n}\right)\frac{b+a}{n}.$$

(D) $$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{j=1}^{n} f\left(a - j\frac{b-a}{n}\right)\frac{b-a}{n}.$$

2. Let $h(x) = f(x) + g(x)$ where f and g are continuous functions. Show that

$$\int_a^b h(x)\,dx = \int_a^b f(x)\,dx + \int_a^b g(x)\,dx$$

by representing the integral as a limit of a sum of areas of approximating rectangles.

3. Find a formula for each of the following sums.

(i) $\sum_{k=1}^{n}(k^2+2^k)$

(ii) $\sum_{k=1}^{n}\left(\frac{k}{n}-1\right)\left(\frac{k}{n}+2\right)$

4. State the following integration formula:

$\int_a^b \sin x\,dx =$ ☐ $\qquad \int_a^b \cos x\,dx =$ ☐

$\int_a^b 7\,dx =$ ☐ $\qquad \int_a^b x\,dx =$ ☐

$\int_a^b x^2\,dx =$ ☐ $\qquad \int_a^b \frac{1}{x}\,dx =$ ☐

assuming $0 < a < b$.