Math 181 Quiz 2 Version A

1. Find all values of x which satisfy

(i) $|x-7| < 2$.

$-2 < x-7 < 2$

$5 < x < 9$

Solution $(5, 9)$.

(ii) $\dfrac{1}{x+5} \geq \dfrac{1}{2x+3}$.

$$\frac{(2x+3)-(x+5)}{(x+5)(2x+3)} \geq 0$$

$$\frac{x-2}{(x+5)(2x+3)} \geq 0$$

$x = 2, -5, -3/2$

	-5	$-3/2$	2	
$x-2$	$-$	$-$	$-$	$+$
$x+5$	$-$	$+$	$+$	$+$
$2x+3$	$-$	$-$	$+$	$+$
	$-$	$+$	$-$	$+$

Solution $(-5, -3/2) \cup [2, \infty)$

2. Suppose $a > 0$, $b > 0$ and $a + b < \pi/2$. Prove $\sin(a+b) = \sin a \cos b + \cos a \sin b$.

Now $\sin(a+b) = h + k$

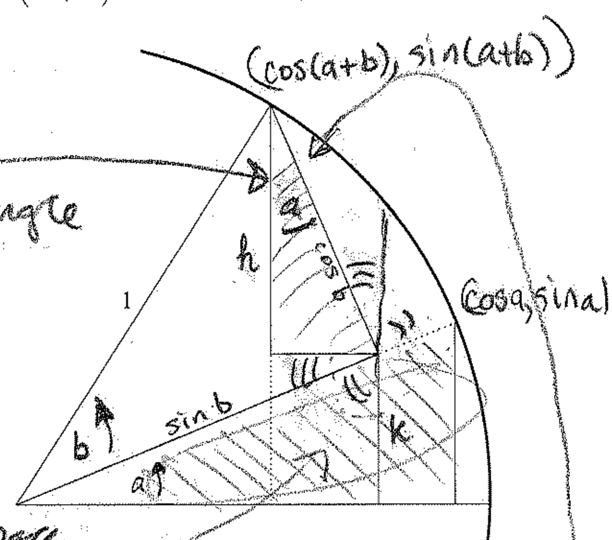
Then $\sin a = \dfrac{h}{\cos b}$ ← This triangle

and $\cos a = \dfrac{k}{\sin b}$ ← This triangle

It follows that

$\sin(a+b) = \sin a \cos b$

$+ \cos a \sin b$



Since the angles marked ((are equal the angles marked))) are equal. But since (((is equal a then this angle is also a

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3. Simplify the the following sums.

(i) $\sum_{k=3}^{n} k = 3+4+5+\cdots+n.$ $= \sum_{k=1}^{n} k - 1 - 2$

$= \frac{n(n+1)}{2} - 3 = \frac{n(n-1)-6}{2}$

$= \frac{n^2-n-6}{2} = \frac{(n-3)(n+2)}{2}$

(ii) $\sum_{k=5}^{25} x^k = x^5+x^6+x^7+\cdots+x^{25}$ under the assumption $x \neq 1$.

$S = x^5 + x^6 + \dots + x^{25}$

$xS = x^6 + x^7 + \dots + x^{26}$

$(1-x)S = x^5 - x^{25}$

Therefore $S = \frac{x^5 - x^{25}}{1-x}$.

4. Use the ϵ-δ definition of contintity to show that $f(x) = 3x$ is continuous at $x_0 = 5$.

Suppose $\varepsilon > 0$.

choose $\delta = \varepsilon/3$.

Then $|x - x_0| < \delta$ implies

$|f(x) - f(5)| = |3x - 3\cdot 5| = 3|x-5|$

$< 3\delta = \varepsilon$.