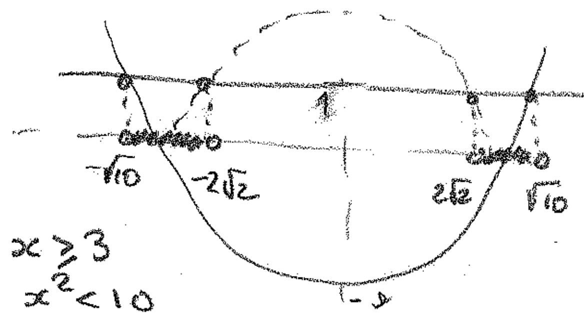
Math 181 Quiz 4 Version A

1. Solve the inequality $|x^2 - 9| < 1$.



Case $x^2 - 9 \geq 0$

Then $x^2 \geq 9$ so either $x \leq -3$ or $x \geq 3$

In this case $x^2 - 9 < 1$ implies $x^2 < 10$

and so $-\sqrt{10} < x < \sqrt{10}$.

Thus x is in $(-\sqrt{10}, \sqrt{10}) \cap ((-\infty, -3] \cup [3, \infty)) = (-\sqrt{10}, -3] \cup [3, \sqrt{10})$

Case $x^2 - 9 < 0$

Then $x^2 < 9$ so $-3 < x < 3$.

In this case $-x^2 + 9 < 1$ implies $x^2 > 8$

and so $x > 2\sqrt{2}$ or $x < -2\sqrt{2}$

Thus x is in $(-3, 3) \cap ((-\infty, -2\sqrt{2}) \cup (2\sqrt{2}, \infty)) = (-3, -2\sqrt{2}) \cup (2\sqrt{2}, 3)$

Combing cases gives $(-\sqrt{10}, -3] \cup (-3, -2\sqrt{2}) \cup (2\sqrt{2}, 3) \cup [3, \sqrt{10}) =$

$$= (-\sqrt{10}, -2\sqrt{2}) \cup (2\sqrt{2}, \sqrt{10})$$

2. Use induction to show that

$$\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \cdots + \frac{1}{n(n+1)} = \frac{n}{n+1}$$

for every positive integer n.

Base case $n = 1$ then $\frac{1}{1 \cdot 2} = \frac{1}{1+1} = \frac{1}{2}$.

Induction step. Suppose $\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \cdots + \frac{1}{n(n+1)} = \frac{n}{n+1}$

Then

$$\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \cdots + \frac{1}{n(n+1)} + \frac{1}{(n+1)(n+2)} = \frac{n}{n+1} + \frac{1}{(n+1)(n+2)}$$

$$= \frac{n(n+2) + 1}{(n+1)(n+2)} = \frac{n^2 + 2n + 1}{(n+1)(n+2)} = \frac{(n+1)^2}{(n+1)(n+2)} = \frac{n+1}{n+2}.$$

Thus completing the induction.

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3. Evaluate the binomial coefficient $\binom{5}{2}$. $= \frac{5!}{2!(5-2)!} = \frac{5\cdot 4}{1\cdot 2} = 10.$

4. State in terms of ε and δ what it means for $f(x)$ to be continuous at x_0.

f is continuous at x_0 if for every $\varepsilon > 0$ there is a $\delta > 0$ such that $|x - x_0| < \delta$ implies $|f(x) - f(x_0)| < \varepsilon$.

5. Prove that if $\lim_{n\to\infty} a_n = L$ and $\lim_{n\to\infty} b_n = M$, then $\lim_{n\to\infty}(a_n + b_n) = L + M$.

Suppose $\varepsilon > 0$.

Let $\varepsilon_1 = \varepsilon/2$. Then by hypothesis there is N_1 such that $n \geq N_1$ implies $|a_n - L| < \varepsilon_1$.

Let $\varepsilon_2 = \varepsilon/2$. Then by hypothesis there is N_2 such that $n \geq N_2$ implies $|b_n - M| < \varepsilon_2$.

Choose $N = \max(N_1, N_2)$. Then $n \geq N$ implies that $n \geq N_1$ and $n \geq N_2$. Therefore

$$|(a_n + b_n) - (L + M)| = |a_n - L + b_n - M|$$

$$\leq |a_n - L| + |b_n - M| < \varepsilon_1 + \varepsilon_2 = \frac{\varepsilon}{2} + \frac{\varepsilon}{2} = \varepsilon.$$