① (i)

$$u'(x) = \lim_{h\to 0} \frac{u(x+h)-u(x)}{h}$$

$$= \lim_{h\to 0} \frac{\sqrt{g(x+h)} - \sqrt{g(x)}}{h}$$

$$= \lim_{h\to 0} \frac{g(x+h) - g(x)}{h\left(\sqrt{g(x+h)} + \sqrt{g(x)}\right)}$$

$$= \underbrace{\lim_{h\to 0} \frac{g(x+h)-g(x)}{h}}_{g'(x)} \cdot \underbrace{\lim_{h\to 0} \frac{1}{\sqrt{g(x+h)}+\sqrt{g(x)}}}_{\frac{1}{2\sqrt{g(x)}}}$$

$$= g'(x) \cdot \frac{1}{2\sqrt{g(x)}}$$

$$= \boxed{\frac{g'(x)}{2\sqrt{g(x)}}}$$

(ii)

$$v'(x) = \lim_{h\to 0} \frac{v(x+h)-v(x)}{h}$$

$$= \lim_{h\to 0} \frac{f((x+h)^2) - f(x^2)}{h}$$

$$= \lim_{h\to 0} \frac{f(x^2+2hx+h^2) - f(x^2)}{h}$$

$$= \underbrace{\lim_{h\to 0} \frac{f(x^2 + h(2x+h)) - f(x^2)}{h}}_{} \cdot \frac{2x+h}{2x+h}$$

let $t = h(2x+h)$

as $h\to 0$, $t\to 0$

$$= \underbrace{\lim_{t\to 0} \frac{f(x^2+t) - f(x^2)}{t}}_{} \cdot \lim_{h\to 0} 2x+h$$

$$= f'(x^2) \cdot 2x$$

$$= \boxed{2x\, f'(x^2)}$$

(iii)

$$w'(x) = \lim_{x\to a} \frac{f(g(x)) - f(g(a))}{x-a} \cdot \frac{g(x)-g(a)}{g(x)-g(a)}$$

$$= \underbrace{\lim_{x\to a} \frac{f(g(x)) - f(g(a))}{g(x)-g(a)}}_{\text{as } x\to a,\ g(x)\to g(a)} \cdot \underbrace{\lim_{x\to a} \frac{g(x)-g(a)}{x-a}}_{g'(x)}$$

$$= \underbrace{\lim_{g(x)\to g(a)} \frac{f(g(x)) - f(g(a))}{g(x)-g(a)}}_{f'(t) \text{ evaluated when } t = g(x)} \cdot g'(x)$$

$$= \boxed{f'(g(x))\, g'(x)}$$

② (i) $f'(x) = \boxed{12x^3 + 3x^2 + 2x + 17}$

(ii)
$$g'(x) = \frac{2x(x^2+1) - (x^2-1)2x}{(x^2+1)^2}$$
$$= \frac{2x(x^2+1-x^2+1)}{(x^2+1)^2}$$
$$= \boxed{\frac{4x}{(x^2+1)^2}}$$

(iii)
$$u'(x) = \frac{d}{dx}(\sin(x))^{-1/2}$$
$$= -\frac{1}{2}\sin(x)^{-3/2} \cdot \frac{d}{dx}\sin(x)$$
$$= -\frac{1}{2}\sin(x)^{-3/2} \cdot \cos(x) = \boxed{-\frac{\cos(x)}{\sqrt{\sin(x)}^{3}}}$$

(iv)
$$v'(x) = \frac{d}{dx}\cos(x^2) = -\sin(x^2)\frac{d}{dx}x^2 = \boxed{-2x\sin(x^2)}$$

(v)
$$w'(x) = \frac{d}{dx}\sin(x-1)\cos(x+1)$$
$$= \frac{d}{dx}(\sin(x+1))\cos(x+1) + \sin(x+1)\frac{d}{dx}\cos(x+1)$$
$$= \cos(x+1)\frac{d}{dx}(x+1)\cos(x+1) - \sin(x+1)\sin(x+1)\frac{d}{dx}(x+1)$$
$$= \boxed{\cos^2(x+1) - \sin^2(x+1)}$$

LANG p. 56

① $\frac{d}{dx} 2x^{\frac{1}{3}} = 2\frac{1}{3}x^{-2/3} = \boxed{\frac{2}{3}x^{-2/3}}$

② $\frac{d}{dx} 5x^{11} = 11 \cdot 5x^{10} = \boxed{55x^{10}}$

⑧ $\frac{d}{dx}(2x^2-1)(x^4+1) = \frac{d}{dx}(2x^2-1)\cdot(x^4+1) + (2x^2-1)\frac{d}{dx}(x^4+1)$

$= 4x(x^4+1) + (2x^2-1)4x^3$

$= 4x(x^4+1+x^2(2x^2-1))$

$= \boxed{4x(3x^4-x^2+1)}$

⑬ $\frac{d}{dx}\frac{2x+1}{x+5} = \frac{2(x+5)-(2x+1)}{(x+5)^2}$

$= \frac{2x+10-2x-1}{(x+5)^2}$

$= \boxed{\frac{9}{(x+5)^2}}$

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③ $\frac{d}{dx}\frac{1}{2}x^{-3/4} = -\frac{3}{4}\cdot\frac{1}{2}x^{-7/4} = \boxed{-\frac{3}{8}x^{-7/4}}$

⑤ $\frac{d}{dx} 25x^{-1} + 12x^{1/2} = (-1)25x^{-2} + \frac{1}{2}\cdot 12x^{-1/2} = \boxed{-25x^{-2}+6x^{-1/2}}$

⑩ $\frac{d}{dx}(2x-5)(3x^4+5x+2) = \frac{d}{dx}(2x-5)\cdot(3x^4+5x+2) + (2x-5)\frac{d}{dx}(3x^4+5x+2)$

$= 2(3x^4+5x+2) + (2x-5)(12x^3+5)$

$= 6x^4+10x+4+24x^4+10x-60x^3-25$

$= \boxed{30x^4-60x^3+20x-21}$

Ayres & Mendelson, p. 87

(27) $\frac{dy}{dx} = \boxed{5x^4 + 20x^3 - 20x}$

(38) $f'(x) = \left(\frac{d}{dx}x\right)\sqrt{3-2x^2} + x\frac{d}{dx}\sqrt{3-2x^2}$

$= \sqrt{3-2x^2} + x \cdot \frac{1}{2\sqrt{3-2x^2}} \frac{d}{dx}(3-2x^2)$

$= \frac{3-2x^2}{\sqrt{3-2x^2}} + \frac{x(-4x)}{2\sqrt{3-2x^2}}$

$= \boxed{\frac{3-4x^2}{\sqrt{3-2x^2}}}$

(40) $\frac{dz}{dw} = \frac{\frac{d}{dw}(w)\left(\sqrt{1-4w^2}\right) - w\frac{d}{dw}\left(\sqrt{1-4w^2}\right)}{1-4w^2}$

$= \frac{\sqrt{1-4w^2} - w \cdot \left(\frac{1}{2}(1-4w^2)^{-1/2}(-8w)\right)}{1-4w^2}$

$= \frac{\sqrt{1-4w^2} + 4w^2(1-4w^2)^{-1/2}}{1-4w^2}$

$= \frac{1-4w^2 + 4w}{(1-4w^2)(1-4w)^{1/2}}$

$= \boxed{\frac{1}{(1-4w^2)^{3/2}}}$

(44) $\frac{ds}{dt} = \frac{2t(3-t^2) - (t^2+2)(-2t)}{(3-t^2)^2}$

$= \frac{6t - 2t^3 + 2t^3 + 4t}{(3-t^2)^2}$

$= \boxed{\frac{10t}{(3-t^2)^2}}$

(55) $$\frac{d^n}{dx^n}(x^{-2}) = \frac{(-1)^n(n+1)!}{x^{2+n}}$$

Proof: By induction. For the base case, note that

$$\frac{d}{dx}(x^{-2}) = -2(x^{-3}) = \frac{(-1)^1(1+1)!}{x^{2+1}}$$

For induction, suppose that

$$\frac{d^{n-1}}{dx^{n-1}}(x^{-2}) = \frac{(-1)^{n-1}(n)!}{x^{2+(n-1)}}.$$

Then

$$\frac{d^n}{dx^n}(x^{-2}) = \frac{d}{dx}\left(\frac{(-1)^{n-1}(n)!}{x^{2+(n-1)}}\right)$$

$$= \frac{(-(2+(n-1))(-1)^{n-1}(n)!}{x^{2+(n-1)+1}} \qquad \text{(power rule)}$$

$$= \frac{(-1)^n(n!)(n+1)}{x^{2+n}}$$

$$= \frac{(-1)^n(n+1)!}{x^{2+n}}.$$

∎

(56) $$\frac{d^n}{dx^n}\left(\frac{1}{3x+2}\right) = \frac{(-1)^n(3)^n(n!)}{(3x+2)^{n+1}}$$

Proof: Again, proof is by induction. Note that $d/dx\,(1/3x+2) = -3/(3x+2)^2$ (by chain rule and power rule), so it holds when $n=1$. Now suppose that

$$\frac{d^{n-1}}{dx^{n-1}}\left(\frac{1}{3x+2}\right) = \frac{(-1)^{n-1}(3)^{n-1}(n-1)!}{(3x+2)^n}$$

Then

$$\frac{d^n}{dx^n}\left(\frac{1}{3x+2}\right) = \frac{d}{dx}\frac{(-1)^{n-1}(3)^{n-1}(n-1)!}{(3x+2)^n}$$

$$= (-1)^{n-1}3^{n-1}(n-1)! \cdot \frac{3(-n)}{(3x+2)^{n+1}}$$

$$= \frac{(-1)^n 3^n (n!)}{(3x+2)^{n+1}}.$$

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