Math 181 Honors Exam 1 Version A

1. Let $a = 0.248$. Find the largest integer n such that $|8a - 2| < 10^{-n}$.

2. Write $3.\overline{25}$ as a fraction of the form p/q where p and q are integers.

3. Sum the infinite series $\displaystyle\sum_{n=2}^{\infty} \frac{1}{5^n}$.

4. Prove one of the following:

(i) Every Cauchy sequence is bounded.

(ii) The harmonic series diverges.

5. State in terms of δ and ϵ what it means for the function $f(x)$ to be continuous at c.

6. Use δ and ϵ to show that $f(x) = 5x$ is continuous at 2.

7. Suppose $g(x)$ is continuous at c and that $g(c) \neq 0$. Use δ and ϵ to show that the function $w(x) = 1/g(x)$ is continuous at c.

8. Determine whether the series

$$\sum_{n=1}^{\infty} \frac{(-1)^n n}{\sqrt{n^3+1}}$$

converges absolutely, converges conditionally or diverges.

9. [Extra Credit] Let $a, b \geq 0$. Prove that $\sqrt{ab} \leq (a+b)/2$.