Math 181 Honors Exam 2 Sample Version A

1. Use the rules of Calculus to find the following derivatives:

(i) $\dfrac{d}{dx}\dfrac{\cos x}{1+|x|}$

(ii) $\dfrac{d}{dx}(7^x x^4)$

(iii) $\dfrac{d}{dx}(2+\sin x)^{(x+1)}$

(iv) $\dfrac{d}{dx}\arctan(1+x^2)$

Math 181 Honors Exam 2 Sample Version A

2. State the definition of the limit

$$\lim_{x \to a} f(x) = L$$

in terms of δ and ϵ.

3. State the definition of the derivative $f'(x)$ in terms of limits.

4. Show that if $f'(x)$ exists at c then $f(x)$ is continuous at c.

5. Suppose $f(x) = 1/x$. Use the limit definition of derivative to show $f'(x) = -1/x^2$.

6. Suppose $w(x) = f(x)g(x)$ where $f(x)$ and $g(x)$ are differentiable. Use the limit definition of derivative to show $w'(x) = f'(x)g(x) + f(x)g'(x)$.

7. Let $a > 0$ and consider the function $f(x) = xe^{-ax}$.

(i) Find $f'(x)$.

(ii) Find ξ such that $f'(\xi) = 0$.

(iii) Show $f(x)$ is increasing on $(-\infty, \xi)$ and decreasing on (ξ, ∞).

(iv) Show that $xe^{-ax} \leq e/a$ for every $x \in \mathbf{R}$.

8. Prove one of the following results:

(i) Linear Approximation Theorem. Let f be twice continuously differentiable on an interval containing a and b. Then there is a point c between a and b such that

$$f(b) = f(a) + f'(a)(b-a) + \frac{f''(c)}{2}(b-a)^2.$$

(ii) Generalized Mean Value Theorem. Suppose f and g are differentiable on (a, b) and continuous on $[a, b]$. If $g'(x) \neq 0$ in (a, b), then there exists a point c in (a, b) such that

$$\frac{f'(c)}{g'(c)} = \frac{f(b) - f(a)}{g(b) - g(a)}.$$