

Math 181 Honors Final Review Sheet/Homework 8

1. Write $1.\overline{87}$ as a fraction of the form p/q where p and q are integers.

2. Sum the infinite series $\sum_{n=-3}^{\infty} \frac{1}{2^n}$.

3. Determine whether the series

$$\sum_{n=1}^{\infty} \frac{(-1)^n n^2}{\sqrt{n+1}}$$

converges absolutely, converges conditionally or diverges.

4. State the definition of the limit

$$\lim_{x \rightarrow a} f(x) = L$$

in terms of δ and ϵ .

5. Use δ and ϵ to show that $f(x) = 3x^2$ is continuous at 1.

6. Suppose $g(x)$ is continuous at 5 Use δ and ϵ to show that the function $w(x) = g(x)/x$ is continuous at 5.

7. Use the rules of Calculus to find the following derivatives:

(i) $\frac{d}{dx} 7^x$

(ii) $\frac{d}{dx} \log \log \log x$

(iii) $\frac{d}{dx} (x \sin 2x)$

(iv) $\frac{d}{dx} \frac{x^2 + 1}{6 + \arctan x}$

8. Suppose $f(x) = 1/x$. Use the limit definition of derivative to show $f'(x) = -1/x^2$.

9. Suppose $w(x) = xf(x)$ where $f(x)$ is differentiable. Use the limit definition of derivative to show $w'(x) = g(x) + xg'(x)$.

10. Find Taylor's formula for $h(x) = \sqrt{1+x}$ when $a = 0$.

11. State Newton's method (or recipe) for finding x such that $f(x) = 0$.

12. Suppose f is three times continuously differentiable. Use Taylor's theorem to show there exists ξ between $a-h$ and $a+h$ such that

$$\frac{f(a+h) - f(a-h)}{2h} = f'(a) + \frac{f'''(\xi)}{6} h^2.$$

13. Find the maximum value of $f(x) = x^2 e^{-x}$ on the interval $[0, 5]$.