Math 181 Final Review Version A

Name: _____ **Recitation:** _____

This answer sheet is the only page you will turn in. Please remove it from the rest of the test and record your answers in the spaces provided.

1.

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5(i). 5(ii). 5(iii).

6(i). 6(ii). 6(iii).

7(i). 7(ii). 7(iii).

8(i). 8(ii). 8(iii).

9(i). 9(ii). 9(iii).

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1. Precisely define $\lim_{x \to a} f(x) = L$ using inequalities in terms of δ and ϵ.

2. Define the derivative $f'(x)$ of a function $f(x)$ using limits.

3. Suppose $x \sin y + y \sin x = 3$. Find dy/dx by implicit differentiation.

4. Define the integral $\int_a^b f(x)dx$ of a function $f(x)$ using limits.

5. Find the following limits:

(i) $\lim_{x \to 2} \dfrac{x^2 - x - 2}{x - 2}$

(ii) $\lim_{x \to \infty} \dfrac{x^2 + x - 3}{2x^2 - 4}$

(iii) $\lim_{t \to 0} \dfrac{1 - e^{-t}}{t}$

6. Find the following derivatives:

(i) $\dfrac{d}{dx} \arctan(2x)$

(ii) $\dfrac{d}{dx}\left(\dfrac{x}{x^2 + 7}\right)$

(iii) $\dfrac{d}{dx}|x|^{3x}$

7. Find the following antiderivatives:

(i) $\displaystyle\int (5x^3 - 2x^2)\, dx$

(ii) $\displaystyle\int x^2 \cos(x^3 + 1)\, dx$

(iii) $\displaystyle\int x\sqrt{x + 1}\, dx$

8. Compute the following areas:

(i) $\displaystyle\int_1^2 x^2 dx$

(ii) $\displaystyle\int_0^{\pi/2} \sin 2x\, dx$

(iii) $\displaystyle\int_1^3 \frac{1}{x + 1}\, dx$

9. Solve the following story problems:

(i) The length of a rectangle is increasing at a rate of 7 cm/s and its width is increasing at a rate of 6 cm/s. When the length is 15 cm and the width is 6 cm, how fast is the area of the rectangle increasing?

(ii) A street light is mounted at the top of a 15-ft-tall pole. A man 6 ft tall walks away from the pole with a speed of 4 ft/s along a straight path. How fast is the tip of his shadow moving when he is 35 ft from the pole?

(iii) A rectangular storage container with an open top is to have a volume of 10 m^3. The length of this base is twice the width. Material for the base costs \$10 per square meter. Material for the sides costs \$6 per square meter. Find the cost of materials for the cheapest such container.

10. Answer the following true/false questions:

(i) If f is differentiable at a, then f is continuous at a.

(ii) If f is continuous on $[a, b]$, then the integral $\int_a^b f(x)dx$ exists.

(iii) $\lim_{\theta \to 0} \dfrac{\sin \theta}{\theta} = 1.$