Math 181 Midterm Practice Version A

1. Precisely define $\lim_{x \to a} f(x) = L$ using inequalities in terms of δ and ϵ.

2. Find the following limits:

(i) $\lim_{x \to 0} \cos x$

(ii) $\lim_{x \to 3} \dfrac{x^2 - 9}{x - 3}$

(iii) $\lim_{x \to \infty} \dfrac{x^2 + x - 3}{2x^2 - 4}$

3. Define the derivative $f'(x)$ of a function $f(x)$ using limits.

4. Use the limit definition to explain why the derivative of $f(x) = 1/x$ is $f'(x) = -1/x^2$.

5. Answer the following true/false questions:

(i) If f is differentiable at a, then f is continuous at a.

(A) True

(B) False

(ii) e is the number such that $\lim_{h \to 0} \dfrac{e^h + 1}{h} = 1$.

(A) True

(B) False

6. Use the facts that

$$\lim_{h\to 0}\frac{\sin h}{h}=1 \qquad \text{and} \qquad \lim_{h\to 0}\frac{(\cos h)-1}{h}=0$$

along with trigonometry to show that $\dfrac{d}{dx}\sin x=\cos x$.

7. Use the rules of calculus and trigonometry to show that $\dfrac{d}{dx}\arcsin x=\dfrac{1}{\sqrt{1-x^2}}$.

8. State the following derivative rules from memory:

$\frac{d}{dx}\sin x =$ $\qquad$ $\frac{d}{dx}(fg)(x) =$

$\frac{d}{dx}\cos x =$ $\qquad$ $\frac{d}{dx}(f \circ g)(x) =$

$\frac{d}{dx}\tan x =$ $\qquad$ $\frac{d}{dx}\left(\frac{f}{g}\right)(x) =$

$\frac{d}{dx}x^{\alpha} =$ $\qquad$ $\frac{d}{dx}\ln x =$ $\qquad$ $\frac{d}{dx}\log_b x =$

$\frac{d}{dx}\arccos x =$ $\qquad$ $\frac{d}{dx}\sec x =$ $\qquad$ $\frac{d}{dx}e^x =$

$\frac{d}{dx}\arcsin x =$ $\qquad$ $\frac{d}{dx}\csc x =$ $\qquad$ $\frac{d}{dx}a^x =$

$\frac{d}{dx}\arctan x =$ $\qquad$ $\frac{d}{dx}\cot x =$ $\qquad$ $\frac{d}{dx}|x| =$

9. Use the rules of calculus to compute the following derivatives:

(i) $\dfrac{d}{dx}(x \sin x)$

(ii) $\dfrac{d}{dx} \arctan(1 + x^2)$

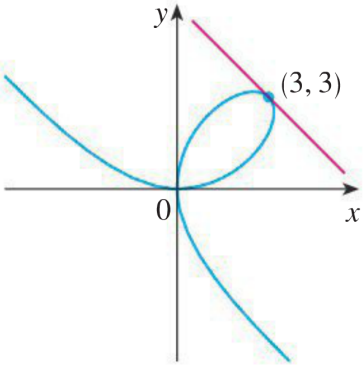
(iii) $\dfrac{d}{dx}\left(\dfrac{x^3 - 5}{x^2 + 4}\right)$

(iv) $\dfrac{d}{dx} x^x$

10. Consider the curve defined by the equation $x^3 + y^3 = 6xy$.

(i) Use implicit differentiation to find y' in terms of x and y.

(ii) Find equation of the line tangent to this curve at the point $(3, 3)$.



(iii) At what point in the first quadrant is the tangent line horizontal?

11. Two carts, A and B, are connected by a rope 39 ft long that passes over a pully P. The point Q is on the floor 12 ft directly beneath P and between the carts. Cart A is being pulled away from Q at a speed of 2 ft/s. How fast is cart B moving toward Q at the instant when cart A is 5 ft from Q?