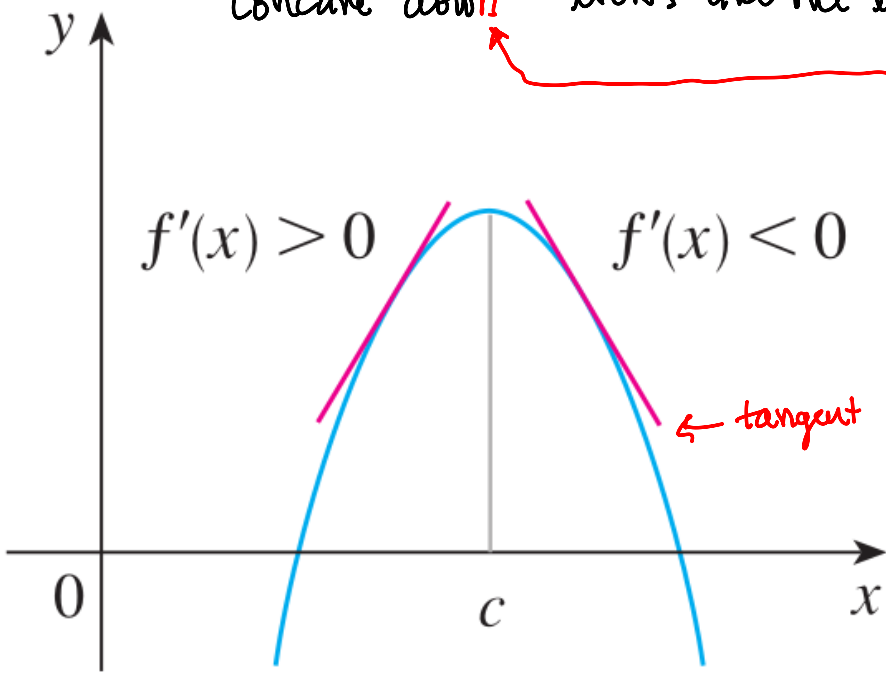
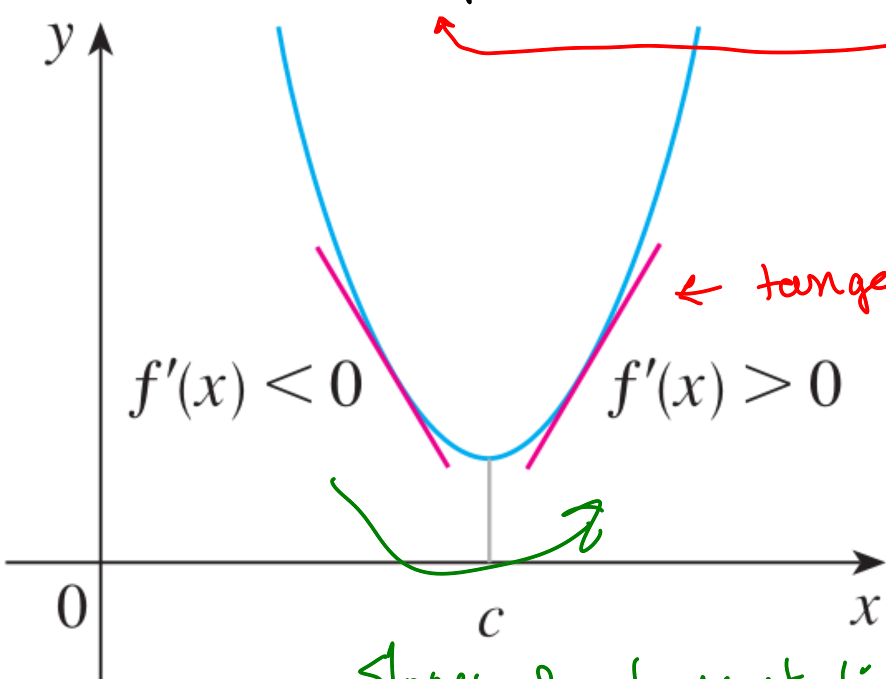
concave down looks like the letter n



← tangent lines lie above the curve...

Slopes of tangent lines decrease, starting from positive slopes, passing through zero and finishing with negative slopes

concave up looks like the letter u



← tangent lines lie below the graphs.

Slopes of tangent lines increase, starting from negative slopes, passing through zero and finishing with positive slopes

• Whether the slopes of the tangents increase or decrease allows characterizing the concavity up or down in terms of the derivative.

Definition If the graph of f lies above all of its tangents on an interval I, then it is called **concave upward** on I. If the graph of f lies below all of its tangents on I, it is called **concave downward** on I.

derivative is decreasing.

derivative of derivative is negative.

then this is decreasing...

derivative increasing

derivative of the derivative is positive.

then this is increasing

Concavity Test

(a) If $f''(x) > 0$ for all x in I, then the graph of f is concave upward on I.

(b) If $f''(x) < 0$ for all x in I, then the graph of f is concave downward on I.

Definition A point P on a curve $y = f(x)$ is called an **inflection point** if f is continuous there and the curve changes from concave upward to concave downward or from concave downward to concave upward at P.

Example

9–18

(a) Find the intervals on which f is increasing or decreasing.

(b) Find the local maximum and minimum values of f.

(c) Find the intervals of concavity and the inflection points.

(d) Draw a sketch of the graph showing the above features

9. $f(x) = x^3 - 3x^2 - 9x + 4$

For (a) need to find where the derivative is + or −.

$$f'(x) = 3x^2 - 6x - 9 + 0 = (3x+3)(x-3) = 0$$

$3x - 9x = -6x$

critical values $x = -1$ or $x = 3$

intervals are $(-\infty, -1)$, $(-1, 3)$ and $(3, \infty)$

For (c) need the second derivative (can only change sign by passing through zero since f' is continuous)

$$f''(x) = \frac{d}{dx}\left(3x^2 - 6x - 9\right) = 6x - 6 = 6(x-1)$$

then $x = 1$ is where $f''(x) = 0$.

intervals are $(-\infty, 1)$ and $(1, \infty)$

Step 1. plot the points where derivate and second derivate is zero and also places where not defined because of jumps or asymptotes.

Step 2. write down the factors of f' or just the one expression for f' if couldn't factor it.

Step 3: multiply the signs to find the sign of f' then infer whether the function is increasing or decreasing

-1 $\quad$ 1 $\quad$ 3

$(3x+3)$	$-$	$+$	$+$	$+$
$(x-3)$	$-$	$-$	$-$	$+$
$f'(x)$	$+$	$-$	$-$	$+$
$f(x)$	$\nearrow$	$\searrow$	$\searrow$	$\nearrow$
$6(x-1)$	$-$	$-$	$+$	$+$
$f''(x)$	$-$	$-$	$+$	$+$
$f(x)$	$\frown$	$\frown$	$\smile$	$\smile$

Step 4 do the same for the second derivative

9. $f(x) = x^3 - 3x^2 - 9x + 4$

maximum $(-1, f(-1)) = (-1, 9)$

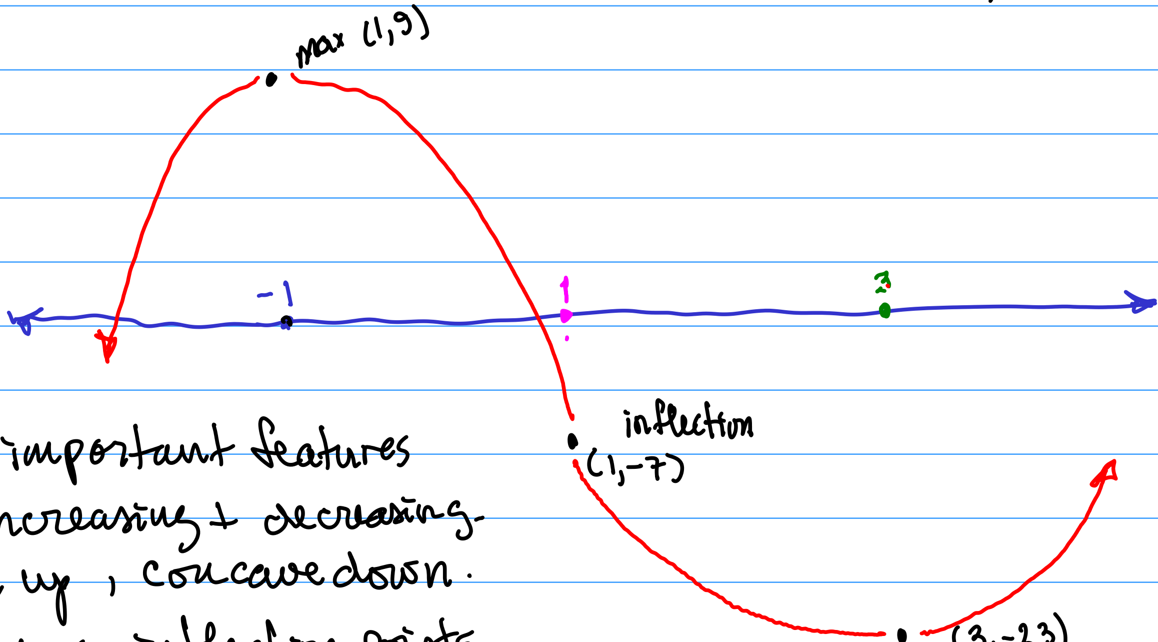
minimum $(3, f(3)) = (3, -23)$

inflution pt $(1, f(1)) = (1, -7)$

$f(-1) = (-1)^3 - 3(-1)^2 - 9(-1) + 4$
$= -1 - 3 + 9 + 4 = 9$

$f(3) = \cancel{3^3} - 3(\cancel{3})^2 - 9(3) + 4$
$= -27 + 4 = -23$

$f(1) = 1^3 - 3(1)^2 - 9(1) + 4$
$= 1 - 3 - 9 + 4 = -7$



shows important features
where increasing + decreasing
concave up, concave down.
min., max, inflection points.
also vertical and horigontal
asymtotes if any..

maximum | inflection point | minimum.

	$(-\infty,-1)$	$(-1,1)$	$(1,3)$	$(3,\infty)$
$(3x+3)$	−	+	+	+
$(x-3)$	−	−	−	+
$f'(x)$	+	−	−	+
$f(x)$	↗	↘	↘	↗
$6(x-1)$	−	−	+	+
$f''(x)$	−	−	+	+
$f(x)$	∩	∩	∪	∪