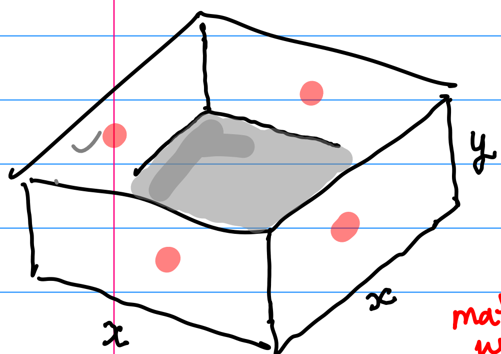
14. A box with a square base and open top must have a volume of 32,000 cm^3. Find the dimensions of the box that minimize the amount of material used.

constant

volume is constant



$$\underbrace{V = x^2 y}_{} = 32000 \text{ cm}^3 = 2^5 10^3 \text{ cm}^3$$

$$y = \frac{V}{x^2}$$

material used

$$M = 4xy + x^2 = \frac{4V}{x} + x^2$$

maximize M on $x \in (0, \infty)$.

Find critical values..

$$\frac{dM}{dx} = \frac{d}{dx}\left(\frac{4V}{x} + x^2\right) = \frac{-4V}{x^2} + 2x = 0$$

power rule

$$\frac{-4V}{x^2} + \frac{2x^3}{x^2} = 0, \quad \frac{2x^3 - 4V}{x^2} = 0 \quad \text{thus } x^3 = 2V, \quad x = \sqrt[3]{2V}$$

skip • Idea check endpoints of the interval and the critical values to find the minimum. Plug into M. ← seemed like more work ... since problem asks for dim. not material

$\boxed{\checkmark}$ • Use derivative information to see where M is increasing and decreasing.

$\sqrt[3]{2V}$ ← where the minimum occurs.

$$M'(x) = \frac{2x^3 - 4V}{x^2} \qquad - \qquad | \qquad +$$

$M(x)$ ↘ ↗

• minimum

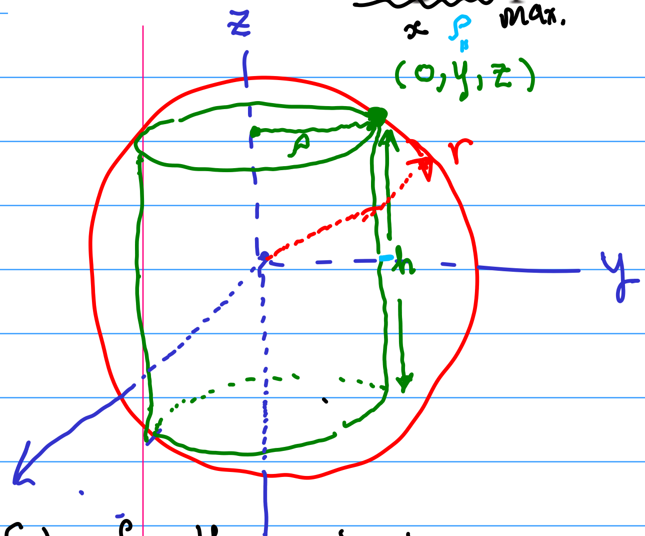
We know $x = \sqrt[3]{2V} = \sqrt[3]{2 \cdot 2^5 10^3 \text{ cm}^3} = \sqrt[3]{2^6 10^3 \text{ cm}^3} = 2^2 10 \text{ cm} = 40 \text{ cm}$

is where the minimum is. Now find y

$$y = \frac{V}{x^2} = \frac{2^5 10^3 \text{ cm}^3}{(2^2 10 \text{ cm})^2} = 2 \cdot 10 \text{ cm} = 20 \text{ cm}.$$

31. A right circular cylinder is inscribed in a sphere of radius r. Find the largest possible volume of such a cylinder.

max.



points on the sphere satisfy

$$x^2 + y^2 + z^2 = r^2$$

relate dimension of cylinder to sphere

$$V = \pi \rho^2 h$$

Note that being inscribed means

$$\tfrac{1}{2} h = z \qquad \rho = y$$

Thus

$$V = \pi y^2 \, 2z = 2\pi y^2 z$$

Solve for the constraint

$$x^2 + y^2 + z^2 = r^2$$

$$0^2 + y^2 + z^2 = r^2$$

$$y^2 = r^2 - z^2$$

$$V = 2\pi (r^2 - z^2) z$$

domain for $z \geq 0$

$$\frac{dV}{dz} = 2\pi (r^2 - 3z^2) = 0$$

don't want negative length

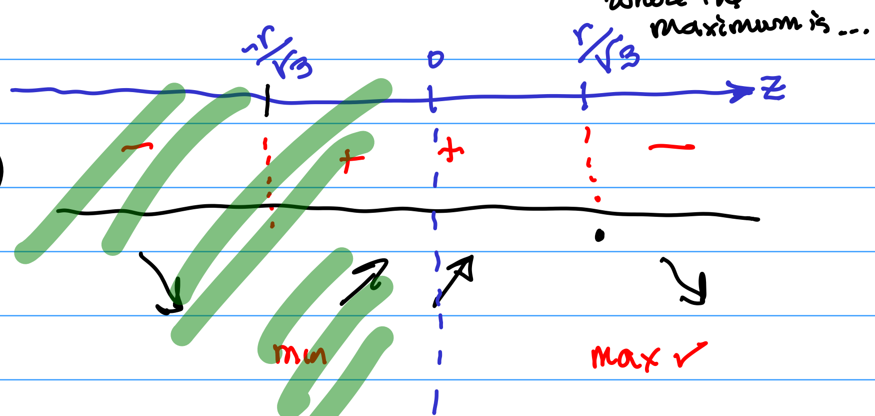
then $r^2 - 3z^2 = 0$, so $z = \pm \frac{r}{\sqrt{3}}$, $z = \frac{r}{\sqrt{3}}$

Check max or min

where the maximum is ...

$V'(z) = 2\pi (r^2 - 3z^2)$

$V(z)$



What is maximum volume? Plug in where the max into V.

$$V\left(\frac{r}{\sqrt{3}}\right) = 2\pi \left(r^2 - \left(\frac{r}{\sqrt{3}}\right)^2\right) \frac{r}{\sqrt{3}} = 2\pi r^2 \left(1 - \frac{1}{3}\right) \frac{r}{\sqrt{3}} = \frac{4\pi}{3\sqrt{3}} r^3.$$

4.9 Antiderivatives

corollary means almost the same as previous theorem...

7 Corollary If $f'(x) = g'(x)$ for all x in an interval (a, b), then $f - g$ is constant on (a, b); that is, $f(x) = g(x) + c$ where c is a constant.

one interval means one constant

Suppose $f'(x) = 1$ what could $f(x)$ be?

$\frac{d}{dx} x = 1$ so $f(x) = \underbrace{x}_{g(x)} + c$

guess g from having worked many problems of differentiation

Suppose $f'(x) = x$ what could $f(x)$ be?

off by a factor of 2

$\frac{d}{dx}\frac{x^2}{2} = \frac{\frac{d}{dx} x^2}{2} = \frac{2x}{2}$ so $f(x) = \frac{x^2}{2} + c$

Continue with antiderivatives on Friday.