Suppose $f'(x) = x^n$ so $f(x) = \frac{1}{n+1}x^{n+1} + c$

Suppose $f'(x) = \ln x$ $f(x) = x\ln x - x + c$

Suppose $f'(x) = \frac{1}{a+x^2}$ $f(x) = \frac{1}{\sqrt{a}} \tan\frac{x}{\sqrt{a}} + c$

Suppose $f'(x) = \frac{1}{x^2+b^2}$ $f(x) = \frac{1}{b}\tan\frac{x}{b} + c$

Suppose $f'(x) = \frac{1}{x^2-b^2}$ need to finish this...

$$\frac{1}{x^2-b^2} = \frac{1}{\underbrace{(x-b)(x+b)}} = \frac{A}{x-b} + \frac{B}{x+b}$$

view this as the common denominator of two fractions

$$\frac{d}{dx}\left(A\ln(x-b) + B\ln(x+b)\right) = \frac{A}{x-b} + \frac{B}{x+b}$$

so $f(x) = A\ln(x-b) + B\ln(x+b) + c$

need to solve for A and B

$A(x+b) + B(x-b) = 1$

$(A+B)x + (A-B)b = 1$

$A = -B$ and $(A-B)b = 1$

$2Ab = 1$

$A = \frac{1}{2b}$

$B = \frac{-1}{2b}$ Thus $f(x) = \frac{1}{2b}\ln(x-b) - \frac{1}{2b}\ln(x+b)$

6. $f(x) = (x-5)^2$

$$\frac{\frac{d}{dx}(x-5)^3}{3} = \frac{3(x-5)^2 \frac{d}{dx}(x-5)}{3} = \frac{3(x-5)^2}{3}$$

$$\frac{d}{dx}\frac{1}{3}(x-5)^3 = (x-5)^2$$

capital letter

arb. constant

The most general antiderivative is $F(x) = (x-5)^2 + c$

22. $f(x) = \dfrac{2x^2+5}{x^2+1} = 2 + \dfrac{3}{x^2+1}$

$$\begin{array}{r} 2 \\ x^2+1 \overline{) 2x^2+5} \\ \underline{2x^2+2} \\ 3 \end{array}$$

$$\frac{d}{dx}2x = 2$$

$$\frac{d}{dx}3\tan x = 3\frac{d}{dx}\tan x = 3\frac{1}{x^2+1}$$

constant.

The most general antiderivative is $F(x) = 2x + 3\tan x + c$

25–48 Find f.

36. $f'(x) = (x+1)/\sqrt{x}$, $f(1) = 5$

information about f that allows to figure out what c is...

find the antiderivative

$$f'(x) = \frac{x+1}{\sqrt{x}} = \frac{x}{\sqrt{x}} + \frac{1}{\sqrt{x}} = x^{1/2} + x^{-1/2}$$

$n = 1/2$ $n = -1/2$

recall

Suppose $f'(x) = x^n$ so $f(x) = \frac{1}{n+1}x^{n+1} + c$

Too many functions called f

$f_1'(x) = x^{1/2}$ $\qquad f_1(x) = \frac{1}{\frac{1}{2}+1} x^{\frac{1}{2}+1} + c_1$

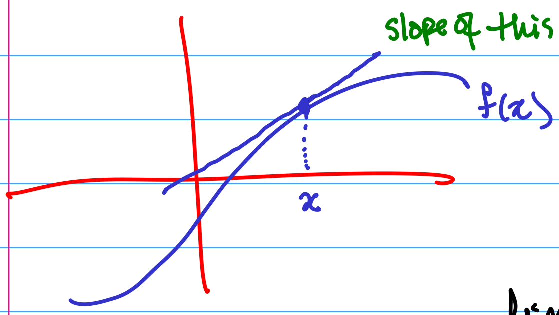
$f_2'(x) = x^{-1/2}$ $\qquad f_2(x) = \frac{1}{-\frac{1}{2}+1} x^{-\frac{1}{2}+1} + c_2$

$$F(x) = \frac{1}{\frac{1}{2}+1} x^{\frac{1}{2}+1} + \frac{1}{-\frac{1}{2}+1} x^{-\frac{1}{2}+1} + c$$

The most general antiderivative is $F(x) = \frac{2}{3} x^{3/2} + 2x^{1/2} + c$

5. Integrals

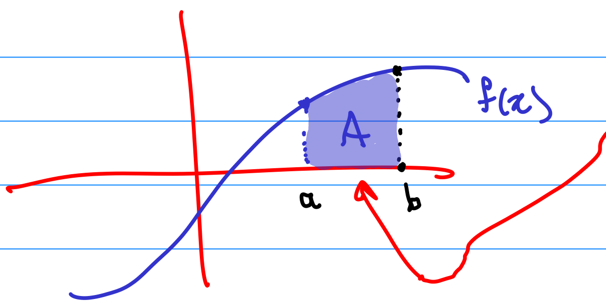
Recall derivatives were slope of tangent lines



$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

limit of the slopes of secant lines which approximate the tangent line..

Integrals are areas under a curve.



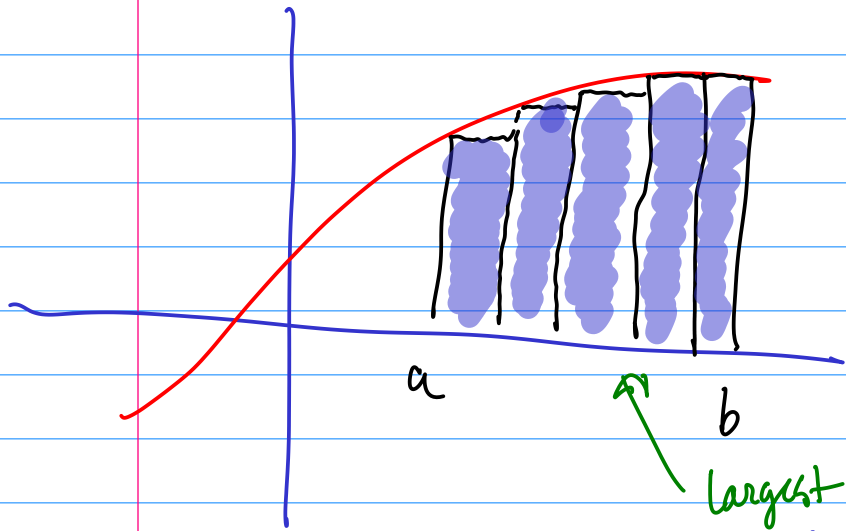
Shaded area is $\qquad A = \int_a^b f(x)\,dx$

Approximate that area and take limits for a mathematical definition....

Need to know derivative table to have good luck when working backwards to find anti derivatives.

$\frac{d}{dx}\sin x =$	$\cos x$	$\frac{d}{dx}(fg)(x) =$	$f'(x)g(x) + f(x)g'(x)$ integration by parts...		
$\frac{d}{dx}\cos x =$	$-\sin x$	$\frac{d}{dx}(f \circ g)(x) =$	$f'(g(x))\, g'(x)$ u-substitution change of vble.		
$\frac{d}{dx}\tan x =$	$\sec^2 x$	$\frac{d}{dx}\left(\frac{f}{g}\right)(x) =$	$\frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$		
$\frac{d}{dx}x^\alpha =$	$\alpha x^{\alpha - 1}$	$\frac{d}{dx}\frac{1}{x^\beta} =$ $x^{-\beta}$	$-\beta \frac{1}{x^{\beta+1}}$	$\frac{d}{dx}\ln x =$	$\frac{1}{x}$
$\frac{d}{dx}\arccos x =$	$\frac{-1}{\sqrt{1-x^2}}$	$\frac{d}{dx}\sec x =$	$\sec x \tan x$	$\frac{d}{dx}\log_b x =$ $\log_b x = \frac{\ln x}{\ln b}$	$\frac{1}{x \ln b}$
$\frac{d}{dx}\arcsin x =$	$\frac{1}{\sqrt{1-x^2}}$	$\frac{d}{dx}\csc x =$	$-\csc x \cot x$	$\frac{d}{dx}e^x =$	e^x
$\frac{d}{dx}\arctan x =$	$\frac{1}{1+x^2}$	$\frac{d}{dx}\cot x =$	$-\csc^2 x$	$\frac{d}{dx}a^x =$ $e^{x \ln a}$	$a^x \ln a$

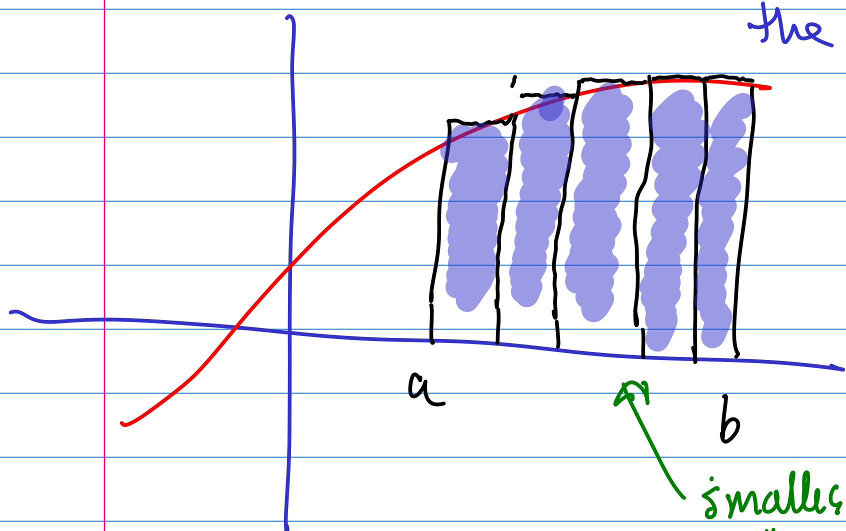
divide area between a and b into rectangles



Idea is add the areas of the rectangles together to obtain an approximation of the area under the curve.

largest rectangles under the curve

This approximation is too small but gets better in the limit as the width of the rectangles get smaller.



smallest rectangles over the curve.

This approximation is too big but gets better in the limit as the width of the rectangles get smaller.

Goal: take the limits and see what you get...

Difficulty: in the limit the widths of rectangles get smaller so the number of rectangles in the sum for the approximation gets larger. In the limit this is a "sum of infinite number of terms".