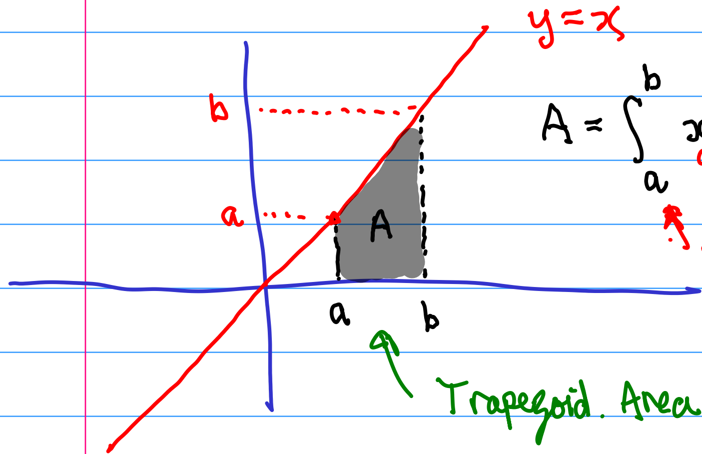
# Examples of areas




$$A = \int_a^b x \, dx$$

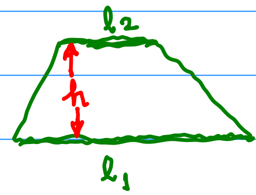
extra decoration, say we are thinking about a function of $x$

under this curve

between $a$ and $b$

Trapezoid. Area $= \frac{1}{2}(l_1 + l_2)\,h$

$a$ $\quad$ $b$ $\quad$ $(b-a)$



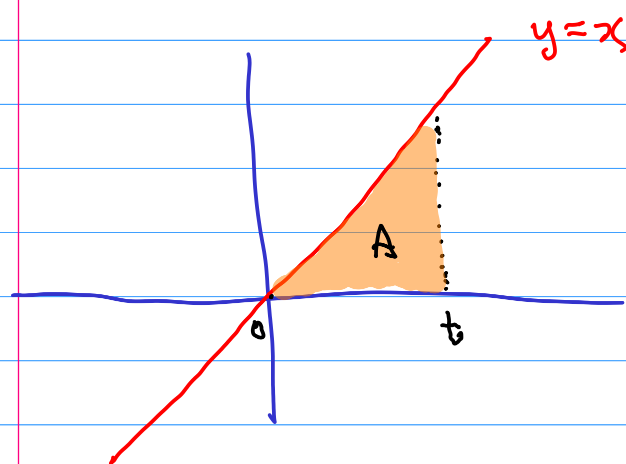

Therefore

factors for a difference of squares

$$A = \int_a^b x \, dx = \frac{1}{2}\underbrace{(a+b)(b-a)}_{} = \frac{1}{2}(b^2 - a^2) = \frac{1}{2}b^2 - \frac{1}{2}a^2$$

Remark $\quad \frac{d}{dx}\frac{1}{2}x^2 = x \qquad$ This was not a coincidence ...

A function defined by an area: $\quad$ above with $a = 0$ and $b = t$




$$A(t) = \int_0^t x \, dx = \frac{1}{2}t^2 - \frac{1}{2}0^2 = \frac{1}{2}t^2$$

$$A'(t) = \frac{d}{dt}\frac{1}{2}t^2 = t$$

not a coincidence

General rule if

$$A(t) = \int_0^t f(x)\,dx \quad \text{then} \quad A'(t) = f(t)$$

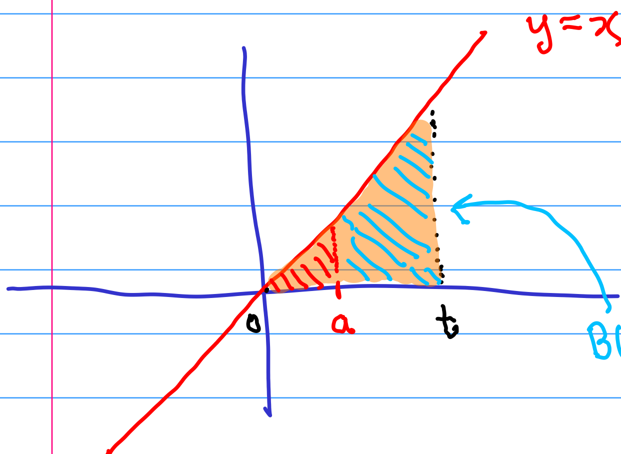
also

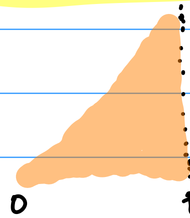
Part of the Fundamental theorem of calculus

$$\boxed{A(t) = \int_a^t f(x)\,dx \quad \text{then} \quad A'(t) = f(t)}$$

constant

about differentiating a function defined by an area... §5.3




$A(t) =$ 


$A(a) =$ 


$$B(t) = \int_a^t x\,dx = A(t) - \underbrace{A(a)}_{\text{constant}} = \frac{1}{2}t^2 - \frac{1}{2}a^2$$

$$B'(t) = A'(t)$$

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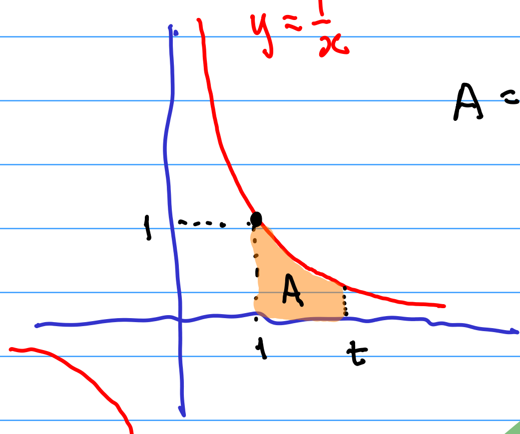
Another example




$$A = \int_1^t \frac{1}{x}\,dx$$

says its a function of $x$.

under the curve $\frac{1}{x}$

area between 1 and $t$

from before

$$A(t) = \int_0^t x\,dx = \frac{1}{2}t^2 - \frac{1}{2}0^2 = \frac{1}{2}t^2$$

$$A'(t) = \frac{d}{dt}\frac{1}{2}t^2 = t$$

not a coincidence

Note $A(t)$ is an antiderivative of $t$

That's the not a coincidence thing mentioned earlier... and called the fundamental theorem of calculus

If $A(t) = \int_1^t \frac{1}{x}\,dx$ then $A'(t) = \frac{1}{t}$

The derivative of the area function is equal the curve that you are finding the area under...

Question: What is the anti-derivative of $\frac{1}{t}$

from the midterm

$$\frac{d}{dx}\ln x = \boxed{\frac{1}{x}}$$

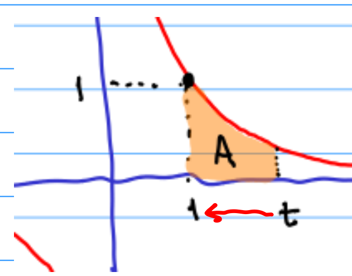
The most general antiderivative

$A(t) = \ln t + c$

constant

figure out constant by looking at the picture

$0 = A(1) = \underbrace{\ln 1}_{0} + c$




thus $c = 0$

$A(1) = 0$

there is no area ever between 1 and 1.

Therefore,

$$\boxed{\int_1^t \frac{1}{x}\,dx = \ln t}$$

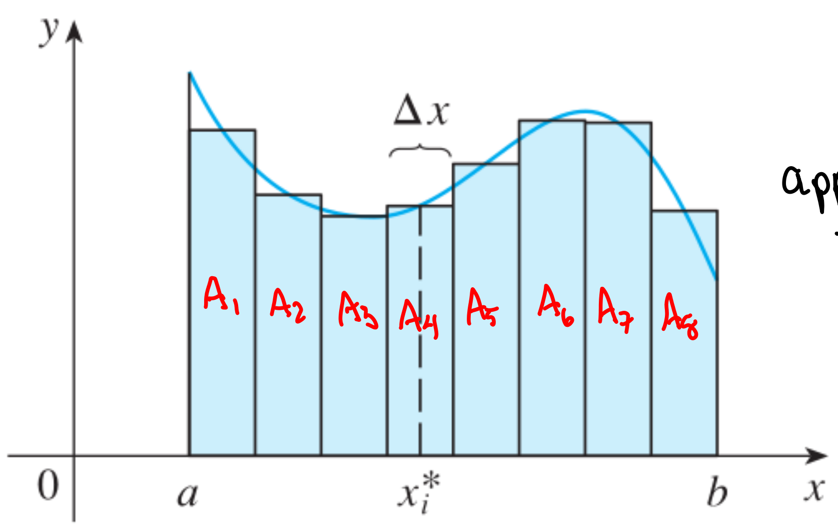
Note: this could be used to define $\ln t$ and avoid the complications of defining $e^x$ first by choosing a base between 2 and 3 for which the tangent at $x=0$ has slope 1.

**2 Definition of a Definite Integral** If $f$ is a function defined for $a \leqslant x \leqslant b$, we divide the interval $[a, b]$ into $n$ subintervals of equal width $\Delta x = (b - a)/n$. We let $x_0\ (= a), x_1, x_2, \ldots, x_n\ (= b)$ be the endpoints of these subintervals and we let $x_1^*, x_2^*, \ldots, x_n^*$ be any **sample points** in these subintervals, so $x_i^*$ lies in the $i$th subinterval $[x_{i-1}, x_i]$. Then the **definite integral of $f$ from $a$ to $b$** is

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\,\Delta x$$

← Σ means a sum...

provided that this limit exists and gives the same value for all possible choices of sample points. If it does exist, we say that $f$ is **integrable** on $[a, b]$.




$$A = \int_a^b f(x)\,dx$$

approx. by the sum of the areas of the rectangles

$$A \approx A_1 + A_2 + \cdots + A_8$$

How to write a sum of a bunch of stuff using a formula...

$$S = 1 + 2 + 3 + \cdots + n = \sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

simplification formula..

Simplify

$$S = n + \cdots + 3 + 2 + 1$$
$$S = 1 + 2 + 3 + \cdots + n$$
$$2S = (n+1) + (n-1+2) + \cdots (1+n)$$

$$2S = n(n+1)$$
$$S = \frac{n(n+1)}{2}$$