How to write a sum of a bunch of stuff using a formula...

$$S = 1+2+3+\cdots+n = \sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

simplification formula..

Simplify

$$S = n + \cdots + 3 + 2 + 1$$
$$S = 1 + 2 + 3 + \cdots + n$$
$$2S = (n+1) + (n-1+2) + \cdots (1+n)$$

$$2S = n(n+1)$$
$$\boxed{S = \frac{n(n+1)}{2}}$$

$n = 5$

$$S = 5 + 4 + 3 + 2 + 1$$
$$S = 1 + 2 + 3 + 4 + 5 \quad \sum_{i=1}^{5} i$$
$$2S = \underbrace{6 + 6 + 6 + 6 + 6}_{5 \text{ term}} = 5 \cdot 6$$

$$\boxed{S = \frac{5 \cdot 6}{2}}$$

$$S = 1^2 + 2^2 + 3^2 + 4^2 + 5^2 \leftarrow \sum_{i=1}^{5} i^2$$

Is there a simplification formula for $\sum_{i=1}^{n} i^2$?

In order to figure out sum of squares consider cubes.

$$\underbrace{-1^3 + 2^3}_{i=1} \underbrace{- 2^3 + 3^3}_{i=2} \underbrace{- 3^3 + 4^3}_{i=3} \underbrace{- 4^3 + 5^3}_{i=4} = -1^3 + 5^3 = 5^3 - 1$$

4 terms

$$\leftarrow \sum_{i=1}^{4} \left(-i^3 + (i+1)^3\right)$$

Therefore

$$\sum_{i=1}^{4}\left(-i^3+(i+1)^3\right) = 5^3-1$$

1
$1\ 1$
$1\ 2\ 1$
$1\ 3\ 3\ 1$

$(i+1)^3 = 1i^3+3i^2+3i+1$

General

$$\sum_{i=1}^{n}\left(-i^3+(i+1)^3\right) = \boxed{(n+1)^3-1}$$

How does this simplification formula for cubes help find a simplification formula for squares? $\sum_{i=1}^{n} i^2$

$$\sum_{i=1}^{n}\left(-i^3+(i+1)^3\right) = \sum_{i=1}^{n}\left(\cancel{-i^3}+\cancel{i^3}+3i^2+3i+1\right)$$

$$= \sum_{i=1}^{n}\left(3i^2+3i+1\right) = 3\sum_{i=1}^{n} i^2 + \underbrace{3\sum_{i=1}^{n} i + \sum_{i=1}^{n} 1}$$

Trying to find a formula for this

I know the simplification formula for these

$$3\sum_{i=1}^{n} i^2 + 3\,\frac{n(n+1)}{2} + n = (n+1)^3-1$$

$-(n+1)$

$$3\sum_{i=1}^{n} i^2 = (n+1)^3 - 3\,\frac{n(n+1)}{2} - n - 1$$

$$= (n+1)\left[(n+1)^2 - \frac{3}{2}n - 1\right]$$

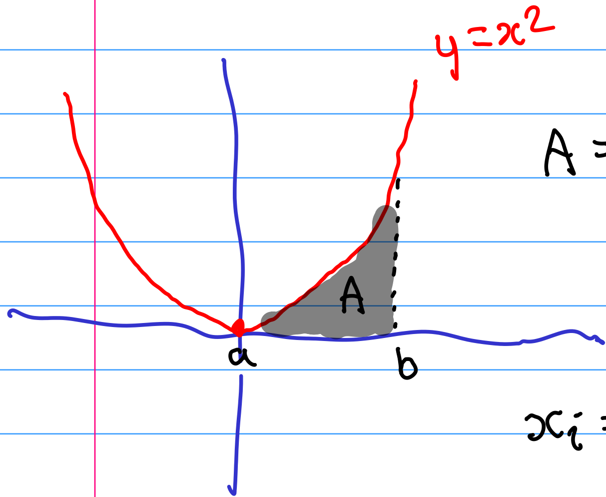
$$= (n+1)\left[n^2+2n\cancel{+1}-\frac{3}{2}n\cancel{-1}\right]$$

$$= n(n+1)\left[n+2-\frac{3}{2}\right] =$$

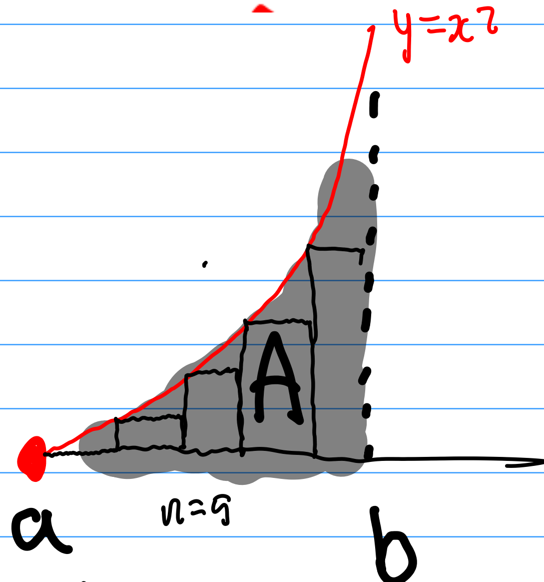
Therefore

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)\left(n+\frac{1}{2}\right)}{3} = \frac{n(n+1)(2n+1)}{6}$$

simplification formula for squares



$$A = \int_a^b x^2\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\,\Delta x$$



$$x_i = a + i\Delta x$$

$$\Delta x = \frac{b-a}{n}$$

$$\lim_{n\to\infty} \sum_{i=1}^{n} x_i^2 \frac{b-a}{n} = \lim_{n\to\infty} \sum_{i=1}^{n} (a+i\Delta x)^2 \frac{b-a}{n}$$

Since $a=0$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left(i\frac{b}{n}\right)^2 \frac{b}{n}$$

$$= \lim_{n\to\infty} \frac{b^3}{n^3} \sum_{i=1}^{n} i^2 = \lim_{n\to\infty} \frac{b^3}{n^3} \frac{n(n+1)(2n+1)}{6}$$

$$= \frac{b^3}{6} \lim_{n\to\infty} 1\cdot\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right) = \frac{b^3}{6}\cdot 2 = \frac{b^3}{3}$$

$$\int_0^b x^2\,dx = \frac{b^3}{3} \qquad \text{recall} \quad \frac{d}{dx}\frac{x^3}{3} = \frac{1}{3}3x^2 = x^2$$

not a coincidence

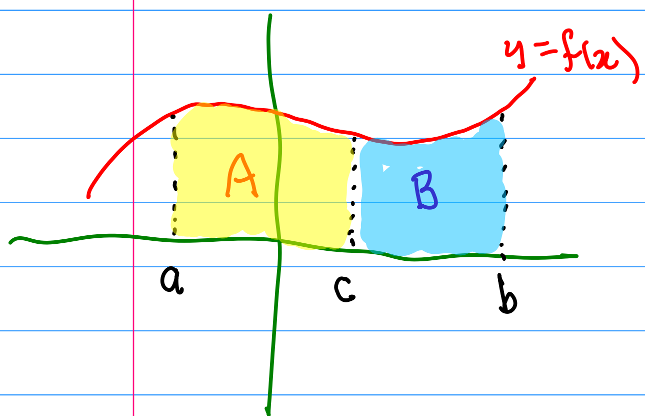
Idea: Show the coincidence is not a coincidence...

Think geometrically.

Areas Add

$$A = \int_a^c f(x)\,dx$$

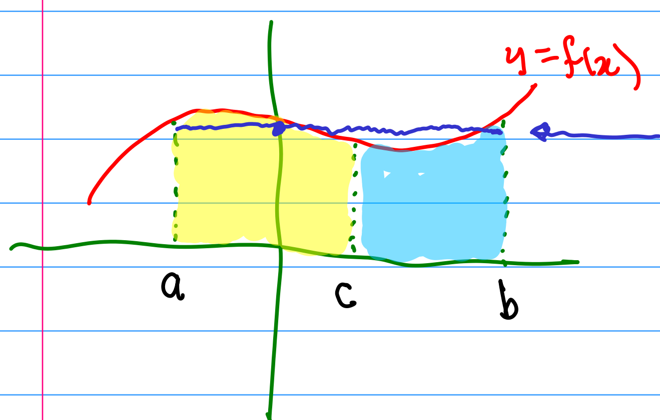
$$B = \int_c^b f(x)\,dx$$



result:

$$\int_a^b f(x)\,dx = A + B = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$$

Average of f

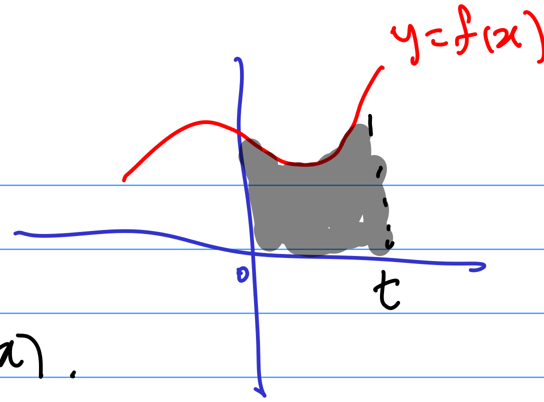


$$\text{average of } f = \frac{1}{b-a}\int_a^b f(x)\,dx$$

area of rectangle is the same as the shaded region.

We want to show something about derivatives and integrals...

$$F(t) = \int_0^t f(x)\,dx$$



want to show $\dfrac{dF(x)}{dx} = f(x)$.

Next time ...