$$\int cf(x)\,dx = c\int f(x)\,dx \qquad \int [f(x)+g(x)]\,dx = \int f(x)\,dx + \int g(x)\,dx$$

$$\int k\,dx = kx + C$$

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C \quad (n \neq -1) \qquad \int \frac{1}{x}\,dx = \ln|x| + C$$

$$\int e^x\,dx = e^x + C \qquad \int b^x\,dx = \frac{b^x}{\ln b} + C$$

$$\int \sin x\,dx = -\cos x + C \qquad \int \cos x\,dx = \sin x + C$$

$$\int \sec^2 x\,dx = \tan x + C \qquad \int \csc^2 x\,dx = -\cot x + C$$

$$\int \sec x \tan x\,dx = \sec x + C \qquad \int \csc x \cot x\,dx = -\csc x + C$$

$$\int \frac{1}{x^2+1}\,dx = \tan^{-1}x + C \qquad \int \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1}x + C$$

$$\int \sinh x\,dx = \cosh x + C \qquad \int \cosh x\,dx = \sinh x + C$$

9. $\int (u+4)(2u+1)\,du$

means find $F(u)$ such that $F'(u) = (u+4)(2u+1)$

Because antiderivatives are used for calculating areas, then similar notation is used for antiderivatives.

Antiderivative

$$\frac{d}{du}\,? = (u+4)(2u+1)$$

trying to find ?

$$? = \int (u+4)(2u+1)\,du$$

indefinite integral... not actually specify the area

the meaning of this notation.

$$\int (u+4)(2u+1)\,du = \int (2u^2 + 9u + 4)\,du = \int 2u^2\,du + \int 9u\,du + \int 4\,du$$

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$$ arbitrary

$$\int 2u^2\,du = 2\int u^2\,du = 2\left(\frac{u^3}{3} + C_1\right)$$

$$\int 9u\,du = 9\int u^1\,du = 9\left(\frac{u^2}{2} + C_2\right)$$

$$\int 4\,du = 4\int u^0\,du = 4\left(\frac{u^1}{1} + C_3\right)$$

Therefore

$$\int (u+4)(2u+1)\,du = 2\left(\frac{u^3}{3} + C_1\right) + 9\left(\frac{u^2}{2} + C_2\right) + 4\left(\frac{u^1}{1} + C_3\right)$$

$$= \frac{2}{3}u^3 + \frac{9}{2}u^2 + 4u + \underbrace{2C_1 + 9C_2 + 4C_3}_{\text{just another arbitrary constant}}$$

$$= \frac{2}{3}u^3 + \frac{9}{2}u^2 + 4u + C$$

12. $\int \left(x^2 + 1 + \frac{1}{x^2+1}\right) dx = \int x^2\,dx + \int 1\,dx + \int \frac{1}{x^2+1}\,dx$

$$\int x^2\,dx = \frac{1}{3}x^3 + C_1$$

$$\int 1\,dx = x + C_2$$

$$\int \frac{1}{x^2+1}\,dx = \arctan x + C_3$$

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$$

$$\int \frac{1}{x^2+1}\,dx = \tan^{-1}x + C$$

$$\int \left(x^2 + 1 + \frac{1}{x^2+1}\right) dx = \frac{1}{3}x^3 + C_1 + x + C_2 + \arctan x + C_3$$

$$= \frac{1}{3}x^3 + x + \arctan x + C$$ just another constant.

15. $\int (2 + \tan^2\theta)\, d\theta$

trigonometry... simplify or transform

$\sin^2\theta + \cos^2\theta = 1$ pythagorean theorem

substitute...

$\int \sec^2 x\, dx = \tan x + C$

$\frac{d \tan x}{dx} = \sec^2 x$

$$\frac{\sin^2\theta}{\cos^2\theta} + \frac{\cos^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$$

$$\tan^2\theta + 1 = \sec^2\theta$$

$$\int (2 + \tan^2\theta)\, d\theta = \int (1 + \tan^2\theta + 1)\, d\theta = \int (1 + \sec^2\theta)\, d\theta$$

$$= \int 1\, d\theta + \int \sec^2\theta\, d\theta = \theta + \tan\theta + C$$

More on... Chain rule for derivatives...

Suppose $F(u)$ has a derivative $F'(u) = f(u)$

and $g(x)$ has a derivative $g'(x)$

Then $\frac{d}{dx} F(g(x)) = \underbrace{F'(g(x))\, g'(x)}_{\text{chain rule}} = f(g(x))\, g'(x)$

So far we have The Fundamental Theorem of Calculus (part 1)

If $F(t) = \int_a^t f(x)\, dx$ then $F'(t) = f(t)$

Fundamental Theorem of Calculus (part 2).

Lim: If $\boxed{F(t)}$ is an antiderivative of $f(t)$

then $\int_a^b f(t)\, dt = F(b) - F(a) = F(t)\Big|_a^b$

Substituted $F(g(x))$ into FTC part 2.

$$\frac{d}{dx}F(g(x)) = \overbrace{F'(g(x))\,g'(x)}^{\text{chain r}} = f(g(x))\,g'(x)$$

$$\int_a^b f(g(x))g'(x)\,dx = F(g(b)) - F(g(a))$$

$$= F(g(x))\Big|_{x=a}^{b} = F(u)\Big|_{u=g(a)}^{g(b)} = \int_{g(a)}^{g(b)} f(u)\,du$$

Therefore:

$$\boxed{\int_a^b f(g(x))g'(x)\,dx = \int_{g(a)}^{g(b)} f(u)\,du}$$

chain rule written in terms of how it affects areas...

called u-substitution... but also just change of variables in an integral.

missing...

$$\int_0^{\pi/6} \sin 2x\,dx = \int_0^{\pi/6} f(g(x))\,dx = \frac{2}{2}\int_0^{\pi/6} f(g(x))\,dx$$

$f(u) = \sin u$ $\quad g(x) = 2x$ $\quad g(0) = 2\cdot 0 = 0$ $\quad g(\pi/6) = 2\pi/6 = \pi/3$

find $g'(x)$ and try to put it in...

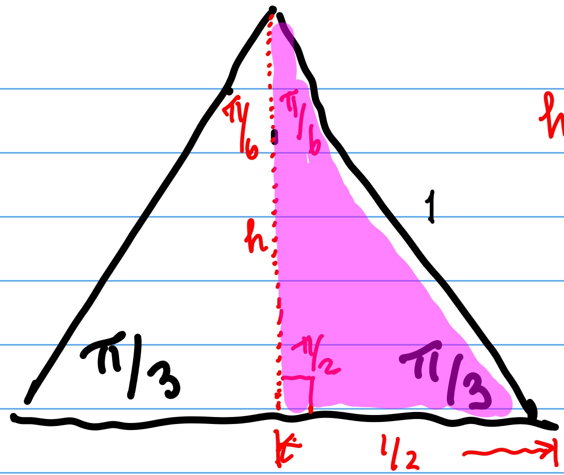
$$g'(x) = \frac{d}{dx}2x = 2$$

$$= \frac{1}{2}\int_0^{\pi/6} f(g(x))\,2\,dx = \frac{1}{2}\int_0^{\pi/6} f(g(x))g'(x)\,dx = \frac{1}{2}\int_{g(0)}^{g(\pi/6)} f(u)\,du$$

$$= \frac{1}{2}\int_0^{\pi/3} \sin u\,du = \frac{1}{2}\left(-\cos u\Big|_0^{\pi/3}\right) = \frac{1}{2}\left(-\cos \pi/3 + \cos 0\right)$$

$$\int \sin x\,dx = -\cos x + C$$

$$= \frac{1}{2}\left(-\frac{1}{2}+1\right) = \frac{1}{4}$$



$$h^2 + \left(\frac{1}{2}\right)^2 = 1^2$$

$$h^2 = \frac{3}{4}$$

$$h = \frac{\sqrt{3}}{2}$$

$$\cos\frac{\pi}{3} = \frac{1/2}{1} = \frac{1}{2}$$

$$\sin\frac{\pi}{3} = \frac{\sqrt{3}/2}{1} = \frac{\sqrt{3}}{2}$$