Math 181 Final Review Version E

Name: _____ **Recitation:** _____

This answer sheet is the only page you will turn in. Please remove it from the rest of the test and record your answers in the spaces provided.

1.

2.

3.

4.

5(i). 5(ii). 5(iii).

6(i). 6(ii). 6(iii).

7(i). 7(ii). 7(iii).

8(i). 8(ii). 8(iii).

9(i). 9(ii). 9(iii).

10(i). (T) (F) 10(ii). (T) (F) 10(iii). (T) (F)

11. Let $g(x) = 1/x$. Use the limit definition of derivative to explain why $g'(x) = -1/x^2$.

12. Use the summation formulas

$$\sum_{k=0}^{n-1} 1 = n, \qquad \sum_{k=0}^{n-1} k = \frac{n(n-1)}{2}, \qquad \sum_{k=0}^{n-1} k^2 = \frac{n(2n-1)(n-1)}{6}$$

and the definition of the definite integral as a limit of sums of approximating rectangles to explain why

$$\int_0^t x^2\,dx = \frac{t^3}{3}.$$

Math 181 Final Review Version E

1. Precisely define $\lim_{x \to a} f(x) = L$ using inequalities in terms of δ and ϵ.

2. Define the derivative $f'(x)$ of a function $f(x)$ using limits.

3. Suppose $2x \sin y + y \sin x = 3$. Find dy/dx by implicit differentiation.

4. Define the integral $\int_a^b f(x)dx$ of a function $f(x)$ using limits.

5. Find the following limits:

(i) $\lim_{x \to 3} \dfrac{x^2 - x - 6}{x - 3}$

(ii) $\lim_{x \to \infty} \dfrac{x^2 + 3x - 1}{3x^2 - 1}$

(iii) $\lim_{t \to 0} \dfrac{1 - e^{-3t}}{t}$

6. Find the following derivatives:

(i) $\dfrac{d}{dx} \arctan(-2x)$

(ii) $\dfrac{d}{dx}\left(\dfrac{x}{x^2 + 5}\right)$

(iii) $\dfrac{d}{dx}|\sin(5 - 2x)|$

7. Find the following antiderivatives:

(i) $\displaystyle\int (3x^3 + x^2)\, dx$

(ii) $\displaystyle\int x^6 \cos(x^7 + 1)\, dx$

(iii) $\displaystyle\int x\sqrt{x - 2}\, dx$

8. Compute the following areas:

(i) $\displaystyle\int_1^2 x^{-3} dx$

(ii) $\displaystyle\int_0^{\pi/2} \sin 2x\, dx$

(iii) $\displaystyle\int_1^4 \frac{1}{x + 2}\, dx$

9. Solve the following story problems:

(i) The length of a rectangle is increasing at a rate of 8 cm/s and its width is increasing at a rate of 5 cm/s. When the length is 12 cm and the width is 4 cm, how fast is the area of the rectangle increasing?

(ii) A street light is mounted at the top of a 12-ft-tall pole. A man 6 ft tall walks away from the pole with a speed of 3 ft/s along a straight path. How fast is the tip of his shadow moving when he is 40 ft from the pole?

(iii) A box with an open top is to be constructed from a single sheet of metal 3 inches long and 2 inches wide by cutting out the corners and folding up the sides.

Find the height of the box with the maximimal volume.

10. Answer the following true/false questions:

(i) $\lim_{\theta \to 0} \dfrac{\cos\theta}{\theta} = 1.$

(ii) If f is continuous at a, then f is differentiable at a.

(iii) If f is continuous on $[a, b]$, then the integral $\int_a^b f(x)dx$ exists.