Math 181 Final Review Version E

Name: ______________________ **Recitation:** ______________________

This answer sheet is the only page you will turn in. Please remove it from the rest of the test and record your answers in the spaces provided.

1. $\lim_{x \to a} f(x) = L$ means for every $\varepsilon > 0$ there is $\delta > 0$ such that $0 < |x-a| < \delta$ then $|f(x) - L| < \varepsilon$.

2. $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

or $f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$

3. $y' = \frac{-2\sin y - y\cos x}{2x\cos y + \sin x}$

4. $\int_a^b f(x)\,dx = \lim_{n \to \infty} \sum_{k=1}^{n} f(x_k^*)\Delta x$ where $\Delta x = \frac{b-a}{n}$

and $x_k^* \in [x_{k-1}, x_k]$ where $x_k = a + k\Delta x$.

5(i). 5(ii). 5(iii).

6(i). 6(ii). 6(iii).

7(i). 7(ii). 7(iii).

8(i). 8(ii). 8(iii).

9(i). 9(ii). 9(iii).

10(i). (T) (F) 10(ii). (T) (F) 10(iii). (T) (F)

11. Let $g(x) = 1/x$. Use the limit definition of derivative to explain why $g'(x) = -1/x^2$.

$$g'(x) = \lim_{h\to 0} \frac{g(x+h)-g(x)}{h} = \lim_{h\to 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h}$$

Simplify to figure out the limit

$$\frac{\frac{1}{x+h} - \frac{1}{x}}{h} = \frac{1}{h}\left(\frac{1}{x+h} - \frac{1}{x}\right) = \frac{1}{h}\,\frac{\cancel{x} - (\cancel{x}+h)}{x(x+h)} = \frac{1}{\cancel{h}}\,\frac{-\cancel{h}}{x(x+h)} = \frac{-1}{x(x+h)}$$

Therefo

$$g'(x) = \lim_{h\to 0} \frac{-1}{x(x+h)} = \frac{-1}{x(x+0)} = -\frac{1}{x^2}$$

12. Use the summation formulas

$$\sum_{k=0}^{n-1} 1 = n, \qquad \sum_{k=0}^{n-1} k = \frac{n(n-1)}{2}, \qquad \sum_{k=0}^{n-1} k^2 = \frac{n(2n-1)(n-1)}{6}$$

and the definition of the definite integral as a limit of sums of approximating rectangles to explain why

$$\int_0^t x^2\,dx = \frac{t^3}{3}.$$

$$\Delta x = \frac{t-0}{n}, \quad x_k = 0 + k\Delta x, \quad f(x) = x^2$$

$$\int_0^t x^2\,dx = \lim_{n\to\infty} \sum_{k=1}^{n} f(x_k^*)\,\Delta x$$

x_k^* any point in $[x_{k-1}, x_k]$

choose $x_k^* = x_{k-1}$

$$\sum_{k=1}^{n} f(x_k^*)\,\Delta x = \sum_{k=1}^{n} (x_{k-1})^2 \frac{t}{n} = \sum_{k=1}^{n} ((k-1)\underbrace{\Delta x}_{t/n})^2 \frac{t}{n} = \sum_{k=1}^{n} (k-1)^2 \frac{t^3}{n^3}$$

shift indices

$$= \sum_{k=0}^{n-1} k^2 \frac{t^3}{n^3} = \frac{n(2n-1)(n-1)}{6}\,\frac{t^3}{n^3} = \frac{\cancel{2}n^3 + \text{lower powers of } n}{\cancel{6}_{3}}\,\frac{t^3}{n^3}$$

$$= \left(\frac{1}{3} + \frac{\text{lower powers of } n}{6n^3}\right)t^3$$

Therefore

$$\lim_{n\to\infty} \sum_{k=1}^{n} f(x_k^*)\,\Delta x = \lim_{n\to\infty}\left(\frac{1}{3} + \cancelto{0}{\frac{\text{lower powers of } n}{6n^3}}\right)t^3 = \frac{1}{3}t^3.$$

12. Use the summation formulas

$$\sum_{k=1}^{n} 1 = n, \qquad \sum_{k=1}^{n} k = \frac{n(n+1)}{2}, \qquad \sum_{k=1}^{n} k^2 = \frac{n(2n+1)(n+1)}{6}$$

and the definition of the definite integral as a limit of sums of approximating rectangles to explain why

$$\int_0^t x^2\,dx = \frac{t^3}{3}.$$

$$\left\{\begin{array}{l} \Delta x = \dfrac{t-0}{n} \\ x_k = 0 + k\Delta x \\ f(x) = x^2 \end{array}\right.$$

$$\int_0^t f(x)\,dx = \lim_{n\to\infty} \sum_{k=1}^{n} f(x_k^*)\Delta x$$

for $x_k^* \in [x_{k-1}, x_k]$

$$\sum_{k=1}^{n} f(x_k^*)\Delta x = \sum_{k=1}^{n} (0+k\Delta x)^2 \Delta x = \sum_{k=1}^{n} k^2 (\Delta x)^3 = \sum_{k=1}^{n} k^2 \left(\frac{t}{n}\right)^3$$

use $x_k^* = x_k$

$$= \frac{n(2n+1)(n+1)}{6}\,\frac{t^3}{n^3} = \frac{\cancel{n}(2n^2+3n+1)}{6}\,\frac{t^3}{n^{\cancel{3}\,2}} = \left(\frac{1}{3} + \frac{1}{2n} + \frac{1}{n^2}\right)t^3$$

Therefore

$$\int_0^t f(x)\,dx = \lim_{n\to\infty} \left(\frac{1}{3} + \frac{1}{2n} + \frac{1}{n^2}\right)t^3 = \frac{1}{3}t^3$$