Math 181 Midterm Practice Version A

1. Define $\lim_{x \to a} f(x) = L$ using inequalities in terms of δ and ϵ.

2. Define the derivate $f'(x)$ of the function $f(x)$ using limits.

3. Define e^x using limits.

4. Find the following limits:

(i) $\lim_{x \to 0} \cos x$

(ii) $\lim_{x \to 3} \dfrac{x^2 - 9}{x - 3}$

(iii) $\lim_{x \to \infty} \dfrac{x^2 + x - 3}{2x^2 - 4}$

(iv) $\lim_{n \to \infty} \left(n - \sqrt{n^2 + 3n} \right)$

5. Use the limit definition to explain why the derivative of $f(x) = 1/x$ is $f'(x) = -1/x^2$.

6. Use limits to explain why every function that is differentiable is also continuous.

7. Suppose $w(x) = f(x)g(x)$. Use the limit definition of derivative to explain why $w'(x) = f'(x)g(x) + f(x)g'(x)$.

8. Use the facts that

$$\lim_{h\to 0} \frac{\sin h}{h} = 1 \qquad \text{and} \qquad \lim_{h\to 0} \frac{(\cos h) - 1}{h} = 0$$

along with trigonometry to show that $\dfrac{d}{dx} \sin x = \cos x$.

9. State the following derivative rules from memory:

$\frac{d}{dx}x^{\alpha} =$ _____ $\frac{d}{dx}\{f(x)g(x)\} =$ _____

$\frac{d}{dx}e^{x} =$ _____ $\frac{d}{dx}f\big(g(x)\big) =$ _____

$\frac{d}{dx}\ln x =$ _____ $\frac{d}{dx}\frac{f(x)}{g(x)} =$ _____

$\frac{d}{dx}|x| =$ _____ $\frac{d}{dx}f^{-1}(x) =$ _____

$\frac{d}{dx}\arctan x =$ _____ $\frac{d}{dx}10^{x} =$ _____ $\frac{d}{dx}\log_{10} x =$ _____

$\frac{d}{dx}\sin x =$ _____ $\frac{d}{dx}\csc x =$ _____ $\frac{d}{dx}\arcsin x =$ _____

$\frac{d}{dx}\cos x =$ _____ $\frac{d}{dx}\sec x =$ _____ $\frac{d}{dx}\arccos x =$ _____

10. Use the rules of calculus to compute the following derivatives:

(i) $\dfrac{d}{dx}(x \sin x)$

(ii) $\dfrac{d}{dx} \arctan(1 + x^2)$

(iii) $\dfrac{d}{dx}\left(\dfrac{x^3 - 5}{x^2 + 4}\right)$

(iv) $\dfrac{d}{dx} x^x$

11. Find dy/dx when $xe^y = x - y$ by implicit differentiation.

12. A man walks along a straight path at 6 ft/s. A searchlight is located on the ground 10 ft from the path and is kept focused on the man. At what rate is the searchlight rotating when the man is 24 ft from the point on the path closest to the searchlight?