

Maxima 5.13.0 <http://maxima.sourceforge.net>
 Using Lisp GNU Common Lisp (GCL) GCL 2.6.7 (aka GCL)
 Distributed under the GNU Public License. See the file COPYING.
 Dedicated to the memory of William Schelter.
 This is a development version of Maxima. The function bug_report()
 provides bug reporting information.

This worksheet provides a simple computation of e by means of the Taylor series expansion. The first 50 terms of the series are summed to obtain 60 digits of precision.

(%i1) `s:=sum(1/n!,n,0,50);`

(%o1)
$$\sum_{n=0}^{50} \frac{1}{n!}$$

(%i2) `e:ev(s,nouns);`

(%o2)
$$\frac{2666905705783137373306341322880702364612402788688346977445977371}{981099780700431549793955102131121575625085211901952000000000000}$$

(%i3) `fpprec:60;`

(%o3) 60

(%i4) `bfloat(e);`

(%o4) $2.71828182845904523536028747135266249775724709369995957496697_B \times 10^0$

(%i5) `bfloat(%e);`

(%o5) $2.71828182845904523536028747135266249775724709369995957496697_B \times 10^0$

The remainder term in the Taylor series is

$$R_{50} = \frac{1}{50!} \int_0^1 (1-t)^{50} e^t dt \leq \frac{3}{50!} \int_0^1 (1-t)^{50} dt$$

which can be used to estimate the error in the previous approximation.

(%i6) `R:3/50!*integrate((1-t)^50,t,0,1);`

(%o6)
$$\frac{1}{517039584429127426741414338823101070354419906672328704000000000000}$$

(%i7) `bfloat(R);`

(%o7) $1.93408789213715180413625335027563735287536500453306933924958_B \times 10^{-66}$

(%i8)