Math 182 Honors Quiz 5 Version A

1. Solve the following definite and indefinite integrals:

(i) $\int \ln \sqrt{x}\, dx = 2\int u \ln u\, du = u^2 \ln u - \int \frac{u^2}{u} du$

$u = \sqrt{x}$ $\quad p = \ln u \quad dp = \frac{1}{u} du$

$u^2 = x$ $\quad dq = 2u\, du \quad q = u^2$

$2u\,du = dx$

$= u^2 \ln u - \int u\,du = u^2 \ln u - \frac{1}{2}u^2 = x \ln\sqrt{x} - \frac{1}{2}x$

(ii) $\int_0^3 x\sqrt{x+1}\, dx = \int_1^4 (u-1)\sqrt{u}\, du$

$u = x+1 \quad x = u-1$

$du = dx$

$= \int_1^4 (u^{3/2} - u^{1/2})\, du$

$= \frac{2}{5}u^{5/2} - \frac{2}{3}u^{3/2} \Big|_1^4$

$= \frac{2}{5}4^{5/2} - \frac{2}{3}4^{3/2} - \frac{2}{5} + \frac{2}{3}$

$= \frac{2^6}{5} - \frac{2^4}{3} - \frac{2}{5} + \frac{2}{3} = \frac{64-2}{5} - \frac{16-2}{3} = \frac{62}{5} - \frac{14}{3}$

(iii) $\int \frac{1}{x^2+4}\, dx = \frac{1}{4}\int \frac{1}{1+(\frac{x}{2})^2} dx$

$u = \frac{x}{2} \quad du = \frac{dx}{2} \quad dx = 2du$

$= \frac{1}{4} \cdot 2 \int \frac{1}{1+u^2} du = \frac{1}{2} \arctan u$

$= \frac{1}{2} \arctan \frac{x}{2}$

Math 182 Honors Quiz 6 Version A

2. State 4 terms of the Taylor series for $f(x) = \dfrac{1}{x+3}$ expanded about $a = 1$.

$$\frac{1}{1-t} = 1 + t + t^2 + t^3 + \cdots \quad \text{for } a = 0.$$

$$\frac{1}{x+3} = \frac{1}{4 + (x-1)} = \frac{1}{4} \cdot \frac{1}{1 + \left(\frac{x-1}{4}\right)} \quad \text{so} \quad t = -\left(\frac{x-1}{4}\right)$$

$$= \frac{1}{4} \cdot \left(1 - \left(\frac{x-1}{4}\right) + \left(\frac{x-1}{4}\right)^2 - \left(\frac{x-1}{4}\right)^3 + \cdots\right)$$

$$= \frac{1}{4} - \frac{1}{4^2}(x-1) + \frac{1}{4^3}(x-1)^2 - \frac{1}{4^4}(x-1)^3 + \cdots$$

for $a = 1$

3. Use Taylor's series to compute $\lim\limits_{x\to 0} \dfrac{x + (x-1)\ln(x+1)}{x^2}$.

$$\ln(x+1) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots$$

$$x\ln(x+1) = x^2 - \frac{x^3}{2} + \frac{x^4}{3} - \frac{x^5}{4} + \cdots$$

$$(x-1)\ln(x+1) = \cancel{-x} + \left(1 + \frac{1}{2}\right)x^2 - \left(\frac{1}{2} + \frac{1}{3}\right)x^3 + \left(\frac{1}{3} + \frac{1}{4}\right)x^4 - \cdots$$

$$\lim_{x\to 0} \frac{x + (x-1)\ln(x+1)}{x^2} = \lim_{x\to 0} \frac{\left(1+\frac{1}{2}\right)\cancel{x^2} - \cdots}{\cancel{x^2}} = \frac{3}{2}.$$