Dear Class,

Here is a quick review of all the trigonometry we've used so far in class that I hope will help with your studying.

Pythagorean Theorem

$$\cos^2 x + \sin^2 x = 1$$

consequences:

$$\frac{\cos^2 x}{\cos^2 x} + \frac{\sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}$$

$$1 + \tan^2 x = \sec^2 x$$

and

$$\frac{\cos^2 x}{\sin^2 x} + \frac{\sin^2 x}{\sin^2 x} = \frac{1}{\sin^2 x}$$

$$\cot^2 x + 1 = \csc^2 x .$$

Summary:

$$\boxed{\begin{array}{l} \cos^2 x + \sin^2 x = 1 \\ 1 + \tan^2 x = \sec^2 x \\ \cot^2 x + 1 = \csc^2 x . \end{array}}$$

Angle addition formula

$$\sin(a+b) = \sin a \cos b + \cos a \sin b$$

Consequences:

$$\frac{d}{da}\sin(a+b) = \frac{d}{da}(\sin a \cos b + \cos a \sin b)$$

$$\cos(a+b) = \cos a \cos b - \sin a \sin b.$$

set $a=b=x$

$$\sin 2x = \sin x \cos x + \cos x \sin x = 2\sin x \cos x.$$

$$\cos 2x = \cos^2 x - \sin^2 x$$

$$= 1-\sin^2 x - \sin^2 x = 1-2\sin^2 x$$

or

$$= \cos^2 x - (1-\cos^2 x) = 2\cos^2 x - 1$$

Solving gives

$$\sin^2 x = \frac{1-\cos 2x}{2} \quad \text{and} \quad \cos^2 x = \frac{1+\cos 2x}{2}.$$

Summary:

$$\boxed{\begin{array}{l} \sin(a+b) = \sin a \cos b + \cos a \sin b \\ \cos(a+b) = \cos a \cos b - \sin a \sin b \\ \sin 2x = 2\sin x \cos x \\ \\ \sin^2 x = \dfrac{1-\cos 2x}{2} \quad \text{and} \quad \cos^2 x = \dfrac{1+\cos 2x}{2}. \end{array}}$$

Product formulas:

$$\sin(a+b) = \sin a \cos b + \cos a \sin b$$
$$\sin(a-b) = \sin a \cos b - \cos a \sin b$$

$$\sin(a+b) + \sin(a-b) = 2 \sin a \cos b$$

therefore

$$\sin a \cos b = \frac{1}{2}\left(\sin(a+b) + \sin(a-b)\right)$$

Similarly

$$\cos(a+b) = \cos a \cos b - \sin a \sin b$$
$$\cos(a-b) = \cos a \cos b + \sin a \sin b$$

$$\cos(a+b) + \cos(a-b) = 2 \cos a \cos b$$
$$\cos(a+b) - \cos(a-b) = -2 \sin a \sin b$$

therefore

$$\cos a \cos b = \frac{1}{2}\left(\cos(a+b) + \cos(a-b)\right)$$
$$\sin a \sin b = \frac{1}{2}\left(\cos(a-b) - \cos(a+b)\right)$$

Summary:

$$\boxed{\begin{aligned} \sin a \cos b &= \tfrac{1}{2}\left(\sin(a+b) + \sin(a-b)\right) \\ \cos a \cos b &= \tfrac{1}{2}\left(\cos(a+b) + \cos(a-b)\right) \\ \sin a \sin b &= \tfrac{1}{2}\left(\cos(a-b) - \cos(a+b)\right) \end{aligned}}$$

There is a similar list of identities for the hyperbolic functions.

$$\sinh t = \frac{e^t - e^t}{2} \quad \text{and} \quad \cosh t = \frac{e^t + e^t}{2}.$$

with inverses

$$\operatorname{arsinh} s = \ln(s + \sqrt{s^2+1})$$

$$\operatorname{arcosh} s = \ln(s + \sqrt{s^2-1})$$

<u>Hyperbolic identities</u>

$$\cosh^2 t - \sinh^2 t = 1$$

$$1 - \tanh^2 t = \operatorname{sech}^2 t$$

$$\coth^2 t - 1 = \operatorname{csch}^2 t$$

$$\sinh(a+b) = \sinh a \cosh b + \cosh a \sinh b$$

$$\cosh(a+b) = \cosh a \cosh b + \sinh a \sinh b$$

$$\sinh 2t = 2 \sinh t \cosh t$$

$$\sinh^2 t = \frac{\cosh 2t - 1}{2}$$

$$\cosh^2 t = \frac{\cosh 2t + 1}{2}$$

All the best!

Eric.