

Maxima 5.21.1 <http://maxima.sourceforge.net>
using Lisp GNU Common Lisp (GCL) GCL 2.6.7 (a.k.a. GCL)
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Dedicated to the memory of William Schelter.
The function bug_report() provides bug reporting information.

(%i19) eq1:L=sqrt(w^2+(R-r)^2);

(%o19) $L = \sqrt{(R-r)^2 + w^2}$

(%i20) eq2:(2*pi-theta)*rho=2*pi*R;

(%o20) $\rho(2\pi - \vartheta) = 2\pi R$

(%i21) eq3:(2*pi-theta)*(rho-L)=2*pi*r;

(%o21) $(2\pi - \vartheta)(\rho - L) = 2\pi r$

(%i22) A:%pi*rho^2-theta/2*rho^2-(%pi*(rho-L)^2-theta/2*(rho-L)^2);

(%o22) $\frac{\vartheta(\rho - L)^2}{2} - \pi(\rho - L)^2 - \frac{\rho^2 \vartheta}{2} + \pi \rho^2$

(%i23) v:solve([eq1,eq2,eq3],[rho,L,theta]);

(%o23)
$$\left[\left[\rho = \frac{R\sqrt{R^2 - 2rR + w^2 + r^2}}{R - r}, L = \sqrt{R^2 - 2rR + w^2 + r^2}, \vartheta = - \frac{(2\pi R - 2\pi r)\sqrt{(R-r)^2 + w^2} - 2\pi R^2 + 4\pi rR - 2\pi w^2 - 2\pi r^2}{R^2 - 2rR + w^2 + r^2} \right] \right]$$

(%i25) A1:subst(v,A);

(%o25)
$$-\pi \left(\frac{R\sqrt{R^2 - 2rR + w^2 + r^2}}{R - r} - \sqrt{R^2 - 2rR + w^2 + r^2} \right)^2 - \frac{\left(\frac{R\sqrt{R^2 - 2rR + w^2 + r^2}}{R - r} - \sqrt{R^2 - 2rR + w^2 + r^2} \right)^2 \left((2\pi R - 2\pi r)\sqrt{(R-r)^2 + w^2} - 2\pi R^2 + 4\pi rR - 2\pi w^2 - 2\pi r^2 \right)}{2(R^2 - 2rR + w^2 + r^2)} + \frac{R^2 \left((2\pi R - 2\pi r)\sqrt{(R-r)^2 + w^2} - 2\pi R^2 + 4\pi rR - 2\pi w^2 - 2\pi r^2 \right)}{2(R-r)^2} + \frac{\pi R^2 (R^2 - 2rR + w^2 + r^2)}{(R-r)^2}$$

(%i26) A2:ratsimp(A1);

(%o26) $(\pi R + \pi r)\sqrt{R^2 - 2rR + w^2 + r^2}$

(%i27) A3:factor(A2);

(%o27) $\pi(R+r)\sqrt{R^2 - 2rR + w^2 + r^2}$

(%i28)