Honors Math 182 Homework 6 Version A

1. Find the following definite and indefinite integrals:

(i) $\displaystyle\int \sin(3\vartheta)\cos(5\vartheta)\,d\vartheta$

(ii) $\displaystyle\int \frac{1}{(\xi^2-4)^{3/2}}\,d\xi$

(iii) $\displaystyle\int_0^{\pi/2} \sin^4\left(\frac{\varphi}{4}\right)d\varphi$

(iv) $\displaystyle\int_2^7 \frac{1}{\zeta^2\sqrt{\zeta^2-1}}\,d\zeta$

2. Find the volume generated by revolving the region bounded by $y = 4 - x^2$ and $y = x + 3$ about the x-axis.

3. Find the volume generated by revolving the region bounded by $y = \sec x$, $y = 0$, $x = -\pi/4$ and $x = \pi/4$ about the x-axis.

4. Find the volume generated by revolving the region bounded by $y = 2^x$, $y = 1$ and $x = 3$ about the x-axis.

5. Find the volume generated by revolving the region bounded by $y = 2 + \sin x$, $y = 0$, $x = \pi$ and $x = 2\pi$ about the y-axis.

6. Find the volume generated by revolving the region bounded by $y = \sqrt{1+\sqrt{x}}$, $y = 0$, $x = 0$ and $x = 4$ about the y-axis.

7. Find the volume generated by revolving the region bounded by $y = \ln x$, $y = x$, $x = 1$ and $x = 4$ about the y-axis.

8. Consider the curve $\big(f(t), g(t)\big)$ given by $f(t) = t^2$ and $g(t) = t - \frac{1}{3}t^3$.

(i) Find the equation of the line tangent to this curve at the point $\big(f(\frac{1}{2}), g(\frac{1}{2})\big)$.

(ii) Find equation of the circle osculating with this curve at the point $\big(f(\frac{1}{2}), g(\frac{1}{2})\big)$.

9. Consider the curve $\big(C(t), S(t)\big)$ given by

$$C(t) = \int_0^t \cos(u^2)\,du \qquad \text{and} \qquad S(t) = \int_0^t \sin(u^2)\,du.$$

(i) Find the unit tangent vector T at any point $\big(C(t), S(t)\big)$ on this curve.

(ii) Find the unit normal vector N at any point $\big(C(t), S(t)\big)$ on this curve.

(iii) Find the curvature κ at any point $\big(C(t), S(t)\big)$ on this curve.