Honors Math 182 Exam 2 Version B

1. Find to 5 digit accuracy the following definite integrals:

(i) $\displaystyle\int_1^2 (1+u)^{1/3}\,du$

(ii) $\displaystyle\int_0^\pi \frac{\cos x + \sin^2 x}{\sin x + \cos^2 x}\,dx$

(iii) $\displaystyle\int_0^1 \frac{1}{\sqrt{1+y+y^2+y^3}}\,dy$

(iv) $\displaystyle\int_1^e (\ln t)^8\,dt$

2. The Taylor's formula for $\cos x$ when $a = 0$ is

$$\cos x = \sum_{k=0}^{n} \frac{(-1)^k}{(2k)!} x^{2k} + R_n(x) \qquad \text{where} \qquad R_n(x) = \frac{(-1)^{n+1}}{(2n+2)!} x^{2n+2} \cos \xi$$

and ξ is some number between 0 and x. Use the inequality $|\cos \xi| \leq 1$ to

(i) Show that $R_n(5) \to 0$ as $n \to \infty$.

(ii) Estimate how large n has to be in order to guarantee $|R_n(5)| \leq 0.5 \times 10^{-4}$.

(iii) Show that $R_2(x) = \mathcal{O}(x^6)$ as $x \to 0$.

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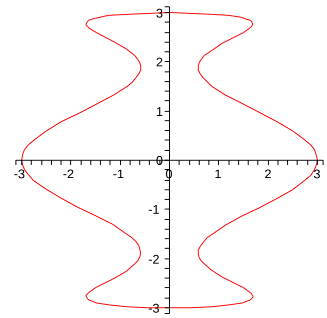
3. Consider the closed curve $(f(t), g(t))$ where $0 \le t \le 2\pi$ given by

$$f(t) = 2\cos t + \cos 5t \qquad \text{and} \qquad g(t) = 3\sin t.$$

(i) Find to 5 digit accuracy the length of this curve.



(ii) Find to 5 digit accuracy the area enclosed by the curve.

(iii) Find the equation of the line tangent to the curve at the point $(\frac{\sqrt{2}}{2}, \frac{3\sqrt{2}}{2})$.

(iv) Find the radius of curvature ρ of the curve at the point $(\frac{\sqrt{2}}{2}, \frac{3\sqrt{2}}{2})$.

4. Suppose $f(x) = \mathcal{O}(x^5)$ and $g(x) = \mathcal{O}(x)$ as $x \to 0$.

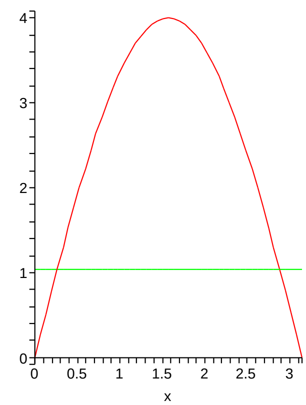
(i) Show $f(x) + g(x) = \mathcal{O}(x)$ as $x \to 0$.

(ii) Show $f(x)g(x) = \mathcal{O}(x^6)$ as $x \to 0$.

5. Find the first 3 non-zero terms of the Taylor series for $\ln(1 + x^2)$ where $a = 0$.

6. Consider the region enclosed by the curve $f(x) = 4\sin x$ and $g(x) = \sqrt{6} - \sqrt{2}$.

(i) Find the volume formed by rotating this region about the x-axis.



(ii) Find the volume formed by rotating this region about the y-axis.

7. Compute the limit $\lim_{x\to 0} \dfrac{\sin x - x\cos x}{x^3}$