Math 285 Exam 1 Version B

1. State Euler's method for solving the ordinary differential equation $y' = f(x, y)$ with the initial condition $y(x_0) = y_0$.

2. Given n let $h = (b - x_0)/n$ and $x_i = x_0 + hi$. The fourth-order Runge-Kutta method for solving $y' = f(x, y)$ such that $y(x_0) = y_0$ is given by

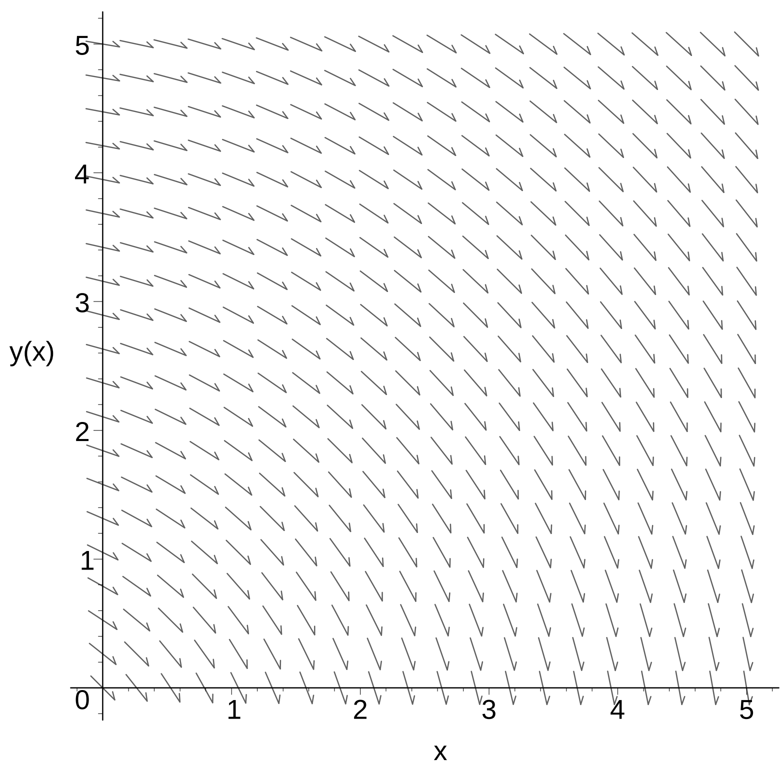
$$\begin{cases} k_1 = hf(x_i, y_i) \\ k_2 = hf(x_i + h/2, y_i + k_1/2) \\ k_3 = hf(x_i + h/2, y_i + k_2/2) \\ k_4 = hf(x_i + h, y_i + k_3) \\ y_{i+1} = y_i + (1/6)(k_1 + 2k_2 + 2k_3 + k_4) \end{cases}$$

Explain what it means in terms of the absolute error $|y(b) - y_n|$ and the step-size h that this method is fourth-order.

3. The direction field for the ordinary differential equation

$$\frac{dy}{dx} = -\frac{1+x}{1+y}$$

is given below. Sketch the solution to the initial value problem $y(0) = 3$ on the direction field.



4. Find the critical points of the autonomous first-order differential equation

$$\frac{dy}{dx} = (y-1)(y-4)^3.$$

Classify each critical point as asymptotically stable, unstable or semi-stable.

5. Find an explicit solution to

$$\frac{dy}{dx} + 7y = 2 - x \qquad \text{where} \qquad y(0) = 1.$$

6. Solve the initial value problem

$$\frac{dy}{dx} = \frac{4x^3+1}{2y+1} \qquad \text{where} \qquad y(0) = 1.$$

Find the exact value of $y(1)$.

7. Consider the matrix

$$A = \begin{bmatrix} -4 & 5 \\ -2 & 2 \end{bmatrix}$$

with eigenvalues and eigenvectors given by

λ	K
$-1+i$	$\begin{bmatrix} 5 \\ 3+i \end{bmatrix}$
$-1-i$	$\begin{bmatrix} 5 \\ 3-i \end{bmatrix}$.

Find the general solution to the system $X' = AX$. Express your answer in terms of sine and cosine functions rather than complex exponentials.

8. Determine whether the differential equation

$$(xe^{yx} + \cos xy)dx + (ye^{xy} - \sin xy)dy = 0$$

is exact.

9. Consider the matrix

$$A = \begin{bmatrix} 2 & 0 & -1 \\ 0 & 2 & -1 \\ -1 & 0 & 2 \end{bmatrix}$$

with exponential

$$e^{At} = \begin{bmatrix} \frac{1}{2}e^{3t} + \frac{1}{2}e^{t} & 0 & -\frac{1}{2}e^{3t} + \frac{1}{2}e^{t} \\ \frac{1}{2}e^{3t} - e^{2t} + \frac{1}{2}e^{t} & e^{2t} & -\frac{1}{2}e^{3t} + \frac{1}{2}e^{t} \\ -\frac{1}{2}e^{3t} + \frac{1}{2}e^{t} & 0 & \frac{1}{2}e^{3t} + \frac{1}{2}e^{t} \end{bmatrix}.$$

Solve the system $X' = AX$ subject to the initial condition $X(0) = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$.

10. Write the system of differential equations

$$\begin{cases} \dfrac{dx}{dt} = 4x + 2y + \sin t \\ \dfrac{dy}{dt} = 2x - 4y + \cos t \end{cases}$$

in the matrix form $X' = AX + F$. What is A? What is F?