

Formula Sheet from Exam 1

Case 1: Distinct Real Roots m_1 and m_2

$$y = c_1 e^{m_1 x} + c_2 e^{m_2 x}.$$

Case 2: Repeated Real Roots $m_1 = m_2$

$$y = c_1 e^{m_1 x} + c_2 x e^{m_1 x}.$$

Case 3: Conjugate Complex Roots $m_1 = \alpha + i\beta$ and $m_2 = \alpha - i\beta$

$$y = e^{\alpha x} (c_1 \cos \beta x + c_2 \sin \beta x).$$

Bernoulli's Equation

$$\frac{dy}{dx} + P(x)y = f(x)y^n.$$

Formula Sheet from Exam 2

The variation of parameters formula for second order ordinary differential equation is

$$y_p = y_1 u_1 + y_2 u_2$$

where

$$u_1 = - \int \frac{y_2 f(t)}{W(y_1, y_2)} dt \quad \text{and} \quad u_2 = \int \frac{y_1 f(t)}{W(y_1, y_2)} dt.$$

The variation of parameters formula for systems is

$$X_p = \Phi(t) \int \Phi^{-1}(t) F(t) dt.$$

Suppose A is a real matrix such that

$$\lambda = \alpha + i\beta \quad \text{and} \quad K = B_1 + iB_2.$$

Then the real solutions corresponding to λ and K are

$$X_1 = [B_1 \cos \beta t - B_2 \sin \beta t] e^{\alpha t}$$

and

$$X_2 = [B_2 \cos \beta t + B_1 \sin \beta t] e^{\alpha t}.$$