1. State Euler’s method for solving the ordinary differential equation \( y' = f(x, y) \) with the initial condition \( y(x_0) = y_0 \).

2. Given \( n \) let \( h = (b - x_0)/n \) and \( x_i = x_0 + hi \). The fourth-order Runge-Kutta method for solving \( y' = f(x, y) \) such that \( y(x_0) = y_0 \) is given by

\[
\begin{align*}
    k_1 &= hf(x_i, y_i) \\
    k_2 &= hf(x_i + h/2, y_i + k_1/2) \\
    k_3 &= hf(x_i + h/2, y_i + k_2/2) \\
    k_4 &= hf(x_i + h, y_i + k_3) \\
    y_{i+1} &= y_i + (1/6)(k_1 + 2k_2 + 2k_3 + k_4)
\end{align*}
\]

Explain what it means in terms of the absolute error \( |y(b) - y_n| \) and the step-size \( h \) that this method is fourth-order.

3. Find the critical points of the autonomous first-order differential equation

\[
\frac{dy}{dx} = (y - 3)(y - 2)^2(y - 1).
\]

Classify each critical point as asymptotically stable, unstable or semi-stable.

4. Find an explicit solution to

\[
\frac{dy}{dx} - 2y = 1 + x \quad \text{where} \quad y(0) = 1.
\]

5. Solve the initial value problem

\[
\frac{dy}{dx} = \frac{3x^2 + 1}{y + 1} \quad \text{where} \quad y(0) = 1.
\]

Find the exact value of \( y(1) \).

6. Determine whether the differential equation

\[
(e^{2y} - y \cos xy)dx + (2xe^{2y} - x \cos xy + 2y)dy = 0
\]

is exact.

7. Solve the initial value problem

\[
ty' + 2y = \sin t \quad \text{where} \quad y(\pi/2) = 1.
\]
Math 285 Exam 1 Version A

8. Solve the initial value problem

\[ y' = xy^3(1 + x^2)^{1/2} \quad \text{where} \quad y(0) = 1. \]

9. Find the critical points of the autonomous first-order differential equation

\[ \frac{dy}{dx} = y^3 - 9y \]

Classify each critical point as asymptotically stable, unstable or semi-stable.

10. Solve the equation

\[ y \, dx + (2x - ye^y) \, dy = 0 \]

Hint: Make it exact by using an integrating factor \( \mu = \mu(y) \).

11. Find the general solution to the second order differential equation \( y'' - 2y' + y = 0 \).

12. Solve the second order initial value problem

\[ y'' - 6y' + 10y = 0 \quad \text{where} \quad y(0) = 1 \quad \text{and} \quad y'(0) = 3. \]

13. Find the general solution to the initial value problem \( y' = \sin(x + y) \).

Hint: Try the substitution \( v = x + y \).

14. Solve the homogeneous initial value problem

\[ (x^2 + 2y^2) \, dx - xy \, dy = 0 \quad \text{where} \quad y(-1) = 1. \]

15. The function \( y_1 = x^4 \) is one solution to the second order differential equation

\[ x^2 y'' - 7xy' + 16y = 0. \]

Find a second linearly independent solution \( y_2 \).

Hint: Substitute \( y_2 = vy_1 \) into the differential equation and then solve for \( v \).

16. State Theorem 1.1 from the text on the existence and uniqueness of solutions to the initial value problem

\[ \frac{dy}{dx} = f(x, y) \quad \text{where} \quad y(x_0) = y_0. \]