TABLE 6.1 Elementary Laplace Transforms

$f(t) = \mathscr{L}^{-1}\{F(s)\}$	$F(s) = \mathscr{L}\{f(t)\}$	Notes
1	$\dfrac{1}{s}, \quad s > 0$	Sec. 6.1; Ex. 4
e^{at}	$\dfrac{1}{s-a}, \quad s > a$	Sec. 6.1; Ex. 5
$\sin at$	$\dfrac{a}{s^2+a^2}, \quad s > 0$	Sec. 6.1; Ex. 6
t^n, $n =$ positive integer	$\dfrac{n!}{s^{n+1}}, \quad s > 0$	Sec. 6.1; Prob. 10
t^p, $p > -1$	$\dfrac{\Gamma(p+1)}{s^{p+1}}, \quad s > 0$	
$\cos at$	$\dfrac{s}{s^2+a^2}, \quad s > 0$	Sec. 6.1; Prob. 3
$\sinh at$	$\dfrac{a}{s^2-a^2}, \quad s > \lvert a \rvert$	Sec. 6.1; Prob. 4
$\cosh at$	$\dfrac{s}{s^2-a^2}, \quad s > \lvert a \rvert$	
$e^{at} \sin bt$	$\dfrac{b}{(s-a)^2+b^2}, \quad s > a$	Sec. 6.1; Prob. 5
$e^{at} \cos bt$	$\dfrac{s-a}{(s-a)^2+b^2}, \quad s > a$	
$t^n e^{at}$, $n =$ positive integer	$\dfrac{n!}{(s-a)^{n+1}}, \quad s > a$	Sec. 6.1; Prob. 6
$u_c(t)$	$\dfrac{e^{-cs}}{s}, \quad s > 0$	Sec. 6.3
$u_c(t)f(t-c)$	$e^{-cs}F(s)$	Sec. 6.3
$e^{ct}f(t)$	$F(s-c)$	Sec. 6.3
$f(ct)$	$\dfrac{1}{c}F\left(\dfrac{s}{c}\right), \quad c > 0$	Sec. 6.3; Prob. 4
$\displaystyle\int_0^t f(t-\tau)g(\tau)\,d\tau$	$F(s)G(s)$	Sec. 6.5
$\delta(t-c)$	e^{-cs}	Sec. 6.4
$f^{(n)}(t)$	$s^nF(s) - s^{n-1}f(0) - \cdots - f^{(n-1)}(0)$	Sec. 6.2
$(-t)^n f(t)$	$F^{(n)}(s)$	Sec. 6.2; Prob. 14

This page intentionally left blank.

Math 285 Final Review Version A

1. Consider the initial value problem

$$\dot{x} = f(x,t) \qquad \text{with} \qquad x(t_0) = x_0.$$

(i) State the existence and uniqueness theorem showing this ordinary differential equation has a unique solution on some open interval I containing t_0.

(ii) State the Runge–Kutta RK4 method for approximating this differential equation on the interval $[t_0, T]$.

(iii) State the definition of the Laplace transform $\mathcal{L}\{f\}$ of a function f.

2. Solve the initial value problem $\dot{x} = \sin^2(3t)$ with $x(0) = 2$.

3. Draw a phase diagram for the autonoumous first-order ordinary differential equation $\dot{x} = x^3 - 4x^2 + 4x$ on the line below. Label the stationary points with a cross $\times$ and draw arrows on the line indicating the direction in which $x(t)$ is changing.

4. Solve the initial value problem $\dot{x} - 2x = t$ with $x(0) = 1$.

5. Find the general solution to $\dfrac{dy}{dx} = \dfrac{x^2 + y^2}{x^2}$.

6. Show that the ordinary differential equation

$$2xy - 9x^2 + (2y + x^2 + 1)y' = 0$$

is exact and find the general solution.

7. Find the general solution to the differential equation

$$xy' - 2y = -x^3y^2.$$

8. Consider the differential equation $\ddot{x} + 5\dot{x} + 6x = \sin 3t$.

(i) Find a particular solution for this differential equation.

(ii) Find the general solution to this differential equation.

(iii) Find the unique solution such that $x(0) = 0$ and $\dot{x}(0) = 2$.

9. Consider the initial value problem

$$y'' - 2y' + 2 = e^{-t} \qquad \text{with} \qquad y(0) = 3, \quad y'(0) = -1.$$

Use Laplace transforms to solve for $Y(s) = \mathcal{L}\{y\}$. Do *not* invert to find y.

10. Find the following inverse Laplace transforms:

(i) $\mathcal{L}^{-1}\left\{\dfrac{e^{-2s}}{s^2 + s - 2}\right\}$

(ii) $\mathcal{L}^{-1}\left\{\dfrac{2s + 1}{4s^2 + 4s + 5}\right\}$