1. Find the general solution to the Bernoulli differential equation
\[ \dot{x} + 2x = tx^3. \]

2. Find the unique solution to the initial value problem
\[ \dot{x} + 2x = tx^3, \quad x(t_0) = x_0 \]
when \( t_0 = 0 \) and \( x_0 = 4 \).

3. Show that the solution found above blows up sometime between \( t = 0.4 \) and \( 0.5 \).

4. Solve \( 8t + 2 = e^{4t} \) to 4 significant digits to find the approximate time of blow up.

5. Use Euler’s method
\[ x_{k+1} = x_k + hf(x_k, t_k) \quad \text{for} \quad k = 0, 1, \ldots, n - 1 \]
with \( h = 0.4/n \) and \( t_k = kh \) to approximate \( x(0.4) \). Compute the error
\[ E_{\text{Euler}} = |x_n - x(0.4)| \]
for values of \( n \) equal to 5, 10, 20, 40 and 80.

6. Use the Runge-Kutta RK4 method
\[
\begin{align*}
    f_1 &= f(x_k, t_k) \\
    f_2 &= f(x_k + \frac{1}{2}hf_1, t_k + \frac{1}{2}h) \\
    f_3 &= f(x_k + \frac{1}{2}hf_2, t_k + \frac{1}{2}h) \\
    f_4 &= f(x_k + hf_3, t_k + h) \\
    x_{k+1} &= x_k + \frac{1}{6}(f_1 + 2f_2 + 2f_3 + f_4)
\end{align*}
\]
to approximate \( x(0.4) \). Compute the error
\[ E_{\text{RK4}} = |x_n - x(0.4)| \]
for values of \( n \) equal to 5, 10, 20, 40 and 80.

7. Comment on the accuracy of the above numerical approximations.

8. [Extra Credit 3] Approximate \( x(0.2) \) where \( x(t) \) is the unique solution to
\[ \dot{x} - 4\sin x = tx^3, \quad x(0) = 4. \]
This solution also blows up at some time \( t > 0 \). Numerically approximate the time of blow up to 4 significant digits.