Math 285 Homework 4 Version A

1. Find the general solution to the Bernoulli differential equation

$$\dot{x} + 2x = tx^3.$$

2. Find the unique solution to the initial value problem

$$\dot{x} + 2x = tx^3, \qquad x(t_0) = x_0$$

when $t_0 = 0$ and $x_0 = 4$.

3. Show that the solution found above blows up sometime between $t = 0.4$ and 0.5.

4. Solve $8t + 2 = e^{4t}$ to 4 significant digits to find the approximate time of blow up.

5. Use Euler's method

$$x_{k+1} = x_k + hf(x_k, t_k) \qquad \text{for} \qquad k = 0, 1, \ldots, n-1$$

with $h = 0.4/n$ and $t_k = kh$ to approximate $x(0.4)$. Compute the error

$$\mathcal{E}_{\text{Euler}} = \big|x_n - x(0.4)\big|$$

for values of n equal to 5, 10, 20, 40 and 80.

6. Use the Runge-Kutta RK4 method

$$\begin{aligned}
f_1 &= f(x_k, t_k)\\
f_2 &= f(x_k + \tfrac{1}{2}hf_1, t_k + \tfrac{1}{2}h)\\
f_3 &= f(x_k + \tfrac{1}{2}hf_2, t_k + \tfrac{1}{2}h)\\
f_4 &= f(x_k + hf_3, t_k + h)\\
x_{k+1} &= x_k + \tfrac{1}{6}h(f_1 + 2f_2 + 2f_3 + f_4)
\end{aligned}$$

to approximate $x(0.4)$. Compute the error

$$\mathcal{E}_{\text{RK4}} = \big|x_n - x(0.4)\big|$$

for values of n equal to 5, 10, 20, 40 and 80.

7. Comment on the accuracy of the above numerical approximations.

8. [Extra Credit 3] Approximate $x(0.2)$ where $x(t)$ is the unique solution to

$$\dot{x} - 4\sin x = tx^3, \qquad x(0) = 4.$$

This solution also blows up at some time $t > 0$. Numerically approximate the time of blow up to 4 significant digits.