1. State the order of the given ordinary differential equations and whether they are linear or non-linear.



2. In 2005 David Lordkipanidze excavated a hominid skull in the Republic of Georgia that was dated to be 1.8 million years old. The half-life of carbon 14 is approximately 5700 years. What percentage of the original carbon 14 atoms are left in the skull?



An artist's rendition of what the original owner of the skull may have looked like. Credit: J.H. Matternes

3. Consider the initial value problem

$$\frac{dx}{dt} = f(x,t)$$
 with $x(t_0) = x_0$.

State the existence and uniqueness theorem which shows this ordinary differential equation has a unique solution on some open interval I containing t_0 .

4. Solve the following initial value problems:

(i)
$$\dot{x} = t$$
 with $x(0) = 3$.

(ii) $\dot{x} = te^t$ with x(0) = 4.

5. Consider the autonoumous first-order ordinary differential equation

$$\frac{dx}{dt} = x^2 - 3x$$

(i) Find the stationary points and determine their stability.

(ii) Draw a phase diagram. Label the stationary points with an cross \times and draw arrows on the line below indicating the direction in which x(t) is changing.

(iii) Find the unique solution to the differential equation that satisfies the initial condition x(0) = 1.

- 6. Consider the differential equation $xy' y = x^2 \sin x$.
 - (i) Find the general solution.

(ii) Find the unique solution that satisfies y(1) = 1.

7. Determine whether the following differential equations are exact.

(i)
$$2x - 1 + (3y + 7)y' = 0$$

(ii)
$$x^3 + y^3 + 3xy^2y' = 0$$

8. Find the general solution to the homogeneous equation

$$x^{2} + y^{2} + (x^{2} - xy)y' = 0.$$

9. Solve the Bernoulli initial value problem

$$y^{1/2}\frac{dy}{dx} + y^{3/2} = 1$$
 with $y(0) = 4$.