This document completes the problem I did not have time to finish in class. We were working Webassign Chapter 8.2 Problem 8, which reads

8. Find the general solution of the system

$$X' = \begin{bmatrix} 3 & -4 & 0 \\ 1 & 0 & 2 \\ 0 & 2 & 3 \end{bmatrix} X$$

We already used Mathematica to find the eigenvectors and eigenvalues of the matrix by typing

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In[1]:= A={{3,-4,0},{1,0,2},{0,2,3}}
Out[1]= {{3, -4, 0}, {1, 0, 2}, {0, 2, 3}}
In[2]:= Eigenvalues[A]
Out[2]= {3, 3, 0}
In[3]:= Eigenvectors[A]
Out[3]= {{-2, 0, 1}, {0, 0, 0}, {-4, -3, 2}}
```

Since there is a repeated eigenvalue and a missing eigenvector, finding three linearly independent solutions is tricky. In particular, we only know the two solutions X_1 and X_3 given by

$$X_1 = (-2, 0, 1)e^{3t} \qquad \text{and} \qquad X_3 = (-4, -3, 2)e^{0t}$$

but not X_2. To find X_2 consider a solution of the form

$$X_2 = (-2, 0, 1)te^{3t} + Pe^{3t}.$$

and solve for P. We didn't have time to solve for P in class. But will do so now. Note, by the product rule, that

$$X_2' = 3(-2, 0, 1)te^{3t} + \big((-2, 0, 1) + 3P\big)e^{3t}$$

and by the property of eigenvectors that

$$AX_2 = 3(-2, 0, 1)te^{3t} + APe^{3t}.$$

Equating $X_2' = AX_2$ and simplifying yields

$$(-2, 0, 1) + 3P = AP \quad \text{or equivalently} \quad (A - 3I)P = (-2, 0, 1).$$

To solve for P substitute for A and subtract 3 from the diagonal to obtain

$$\begin{bmatrix} 0 & -4 & 0 \\ 1 & -3 & 2 \\ 0 & 2 & 0 \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \\ P_3 \end{bmatrix} = \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix}.$$

This was the last equation I wrote on the board in class. To finish solving for P write this matrix equation as the system

$$\begin{cases} \quad\quad\quad -4P_2 = -2 \\ P_1 - 3P_2 + 2P_3 = 0 \\ \quad\quad\quad\quad 2P_2 = 1. \end{cases}$$

The first and last equations are the same and imply $P_2 = 1/2$. Substituting this value into the second equation yields

$$P_1 - \frac{3}{2} + 2P_3 = 0.$$

As this is an under-determined system we may choose P_3 arbitrarily. Upon taking $P_3 = 1$ it follows that

$$P_1 = \frac{3}{2} - 2 = -1/2.$$

Therefore $P = (-1/2, 1/2, 1)$ and consequently

$$X_2 = (-2, 0, 1)te^{3t} + (-1/2, 1/2, 1)e^{3t}.$$

The general solution is $X(t) = X_1C_1 + X_2C_2 + X_3C_3$ which we enter as

8. **1/1 points** | Previous Answers ZillDiffEQModAp10 8.2.025. My Notes Ask Your Teacher

Find the general solution of the given system.

$$X' = \begin{pmatrix} 3 & -4 & 0 \\ 1 & 0 & 2 \\ 0 & 2 & 3 \end{pmatrix} X$$

X(t) = $< -2,0,1 > \cdot e^{3\cdot t} \cdot A + \left(< -2,0,1 > \cdot t + < -\frac{1}{2}, \frac{1}{2}, 1 > \right) \cdot e^{3\cdot t} \cdot B + < -4, -3, 2 > \cdot C$

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