Math 285 Final Supplement Study Questions Version A

1. Use the convolution theorem to evaluate the Laplace transform

$$F(s) = \mathcal{L}\left\{ \int_0^t \tau e^{t-\tau} d\tau \right\}(s).$$

The Laplace transform is

$F(s) =$

2. Suppose

$$A = \begin{bmatrix} 1 & 6 \\ 7 & 11 \\ 10 & 12 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} -6 & 8 & -3 \\ 1 & -3 & 2 \end{bmatrix}$$

Find the products

$AB =$

$BA =$

3. Consider the matrix

$$A = \begin{bmatrix} -1 & 1 & 0 \\ 1 & 3 & 1 \\ 0 & 4 & -1 \end{bmatrix}$$

with eigenvalues

$$\lambda_1 = 4, \qquad \lambda_2 = -2 \qquad \text{and} \qquad \lambda_3 = -1$$

and corresponding eigenvectors

$$K_1 = \begin{bmatrix} 1 \\ 5 \\ 4 \end{bmatrix}, \qquad K_2 = \begin{bmatrix} 1 \\ -1 \\ 4 \end{bmatrix} \qquad \text{and} \qquad K_3 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}.$$

Find the general solution of the matrix differential equation $X' = AX$.

$X(t) =$

4. Consider the matrix

$$A = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}$$

with eigenvalues

$$\lambda_1 = 1 + 2i \qquad \text{and} \qquad \lambda_2 = 1 - 2i$$

and corresponding eigenvectors

$$K_1 = \begin{bmatrix} 1 \\ i \end{bmatrix}, \qquad \text{and} \qquad K_2 = \begin{bmatrix} i \\ 1 \end{bmatrix}.$$

Find the unique solution of the matrix differential equation

$$X' = AX, \qquad X(0) = \begin{bmatrix} 3 \\ 4 \end{bmatrix}.$$

$X(t) =$

5. Consider the matrix

$$A = \begin{bmatrix} 5 & -1 \\ 4 & 1 \end{bmatrix}$$

with an eigenvalue

$$\lambda_1 = 3 \quad \text{of multiplicity} \quad 2$$

and corresponding eigenvector

$$K_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}.$$

Note that this is the case where there is no second eigenvector. Find the general solution of the matrix differential equation $X' = AX$.

$X(t) =$