DEFINITION 1.1.1 Differential Equation

An equation containing the derivatives of one or more unknown functions (or dependent variables), with respect to one or more independent variables, is said to be a **differential equation (DE).**

Classification:

1. By order.
2. By linearity.

y is the function that is defined by the equation: it's the unknown. What y's satisfy this equation?

Order=2 $y'' + 5y' - 4y = e^x$

linear

Linearity YES look to see in y and it's derivates appear linearly or not...

Order=2 $y'' + 5(y')^3 - 4y = e^x$

Linearity No

not linear in y'

also nonlinear

Order=2

Linearity No $y'' + 5\sin(y') - 4y = e^x$

Classification by Linearity An nth-order ordinary differential equation (4) is said to be **linear** if F is linear in $y, y', \ldots, y^{(n)}$. This means that an nth-order ODE is linear when (4) is $a_n(x)y^{(n)} + a_{n-1}(x)y^{(n-1)} + \cdots + a_1(x)y' + a_0(x)y - g(x) = 0$ or

$$a_n(x)\frac{d^ny}{dx^n} + a_{n-1}(x)\frac{d^{n-1}y}{dx^{n-1}} + \cdots + a_1(x)\frac{dy}{dx} + a_0(x)y = g(x). \qquad (6)$$

$$a_n y^{(n)} + a_{n-1} y^{(n-1)} + \cdots + a_1 y' + a_0 y = g(x)$$

$$a_n(x) y^{(n)} + a_{n-1}(x) y^{(n-1)} + \cdots + a_1(x) y' + a_0(x) y = g(x)$$

functions of x

DEFINITION 1.1.2 Solution of an ODE

Any function ϕ, defined on an interval I and possessing at least n derivatives that are continuous on I, which when substituted into an nth-order ordinary differential equation reduces the equation to an identity, is said to be a **solution** of the equation on the interval.

Classical solution: means the nth derivative is continuous...

Note if the $a_0(x), a_1(x), \ldots, a_n(x)$ are continuous, this implies $g(x)$ has to be continuous as well...

Example: $\quad xy' - y = x^2 \sin x$

Solution: $\quad y = cx - x\cos x$

arbitrary constant...

Check solution: $\quad y' = c - \cos x + x \sin x$

$$\cancel{x}\left(c - \cancel{\cos x} + x \sin x\right) - \left(c\cancel{x} - x\cancel{\cos x}\right) = x^2 \sin x$$

$$\cancel{xc} - x\cancel{\cos x} + x^2 \sin x - \cancel{cx} + x\cancel{\cos x} = x^2 \sin x \quad \boxed{\checkmark}$$

solution defined implicitly.

Check that $x^3+y^3 = 3cxy$ satisfies

$$y' = \frac{y(y^3-2x^3)}{x(2y^3-x^3)}$$

differential equation.