


THEOREM 1.2.1 Existence of a Unique Solution

Let R be a rectangular region in the xy -plane defined by $a \leq x \leq b$, $c \leq y \leq d$ that contains the point (x_0, y_0) in its interior. If $f(x, y)$ and $\partial f / \partial y$ are continuous on R , then there exists some interval $I_0: (x_0 - h, x_0 + h)$, $h > 0$, contained in $[a, b]$, and a unique function $y(x)$, defined on I_0 , that is a solution of the initial-value problem (2).

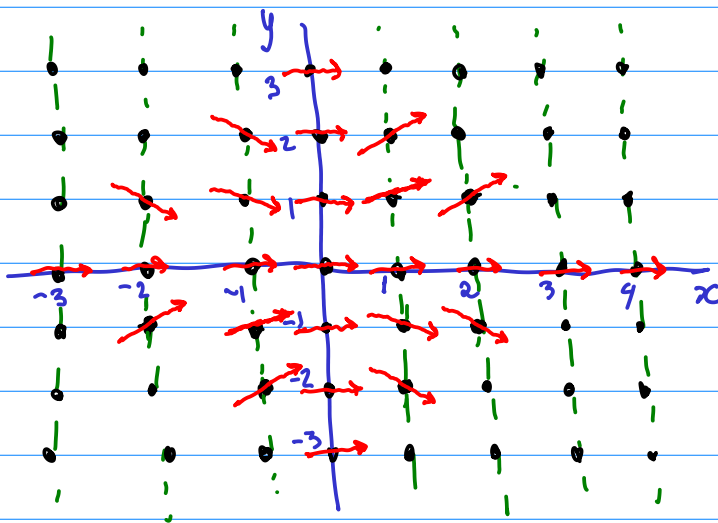
 **Slope** Because a solution $y = y(x)$ of a first-order differential equation

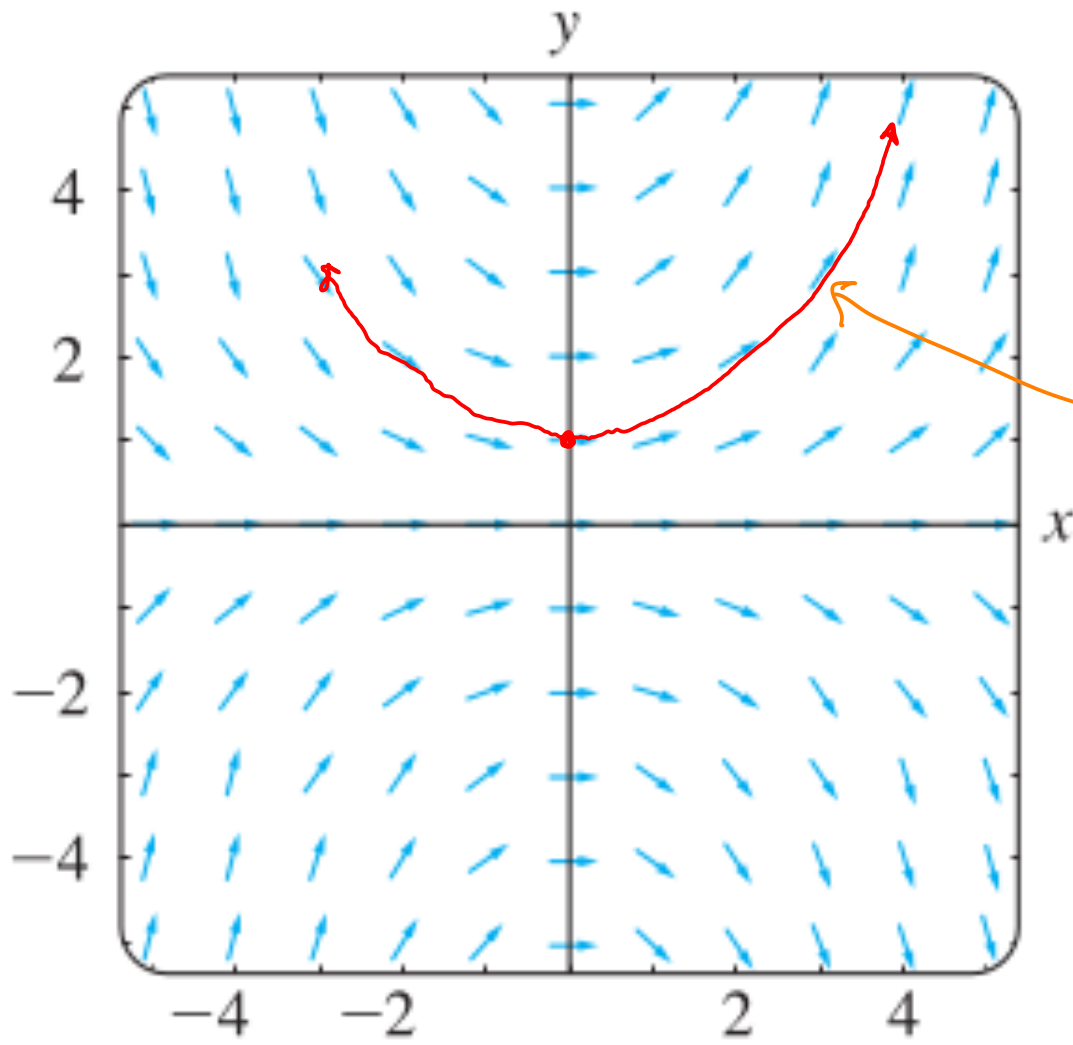
$$\frac{dy}{dx} = f(x, y) \quad (1)$$

Idea sketch a direction field by considering a grid of values for x, y and then drawing a line with that slope at each grid point.

$$\frac{dy}{dx} = \frac{xy}{5}$$

direction field for
 $dy/dx = 0.2xy$

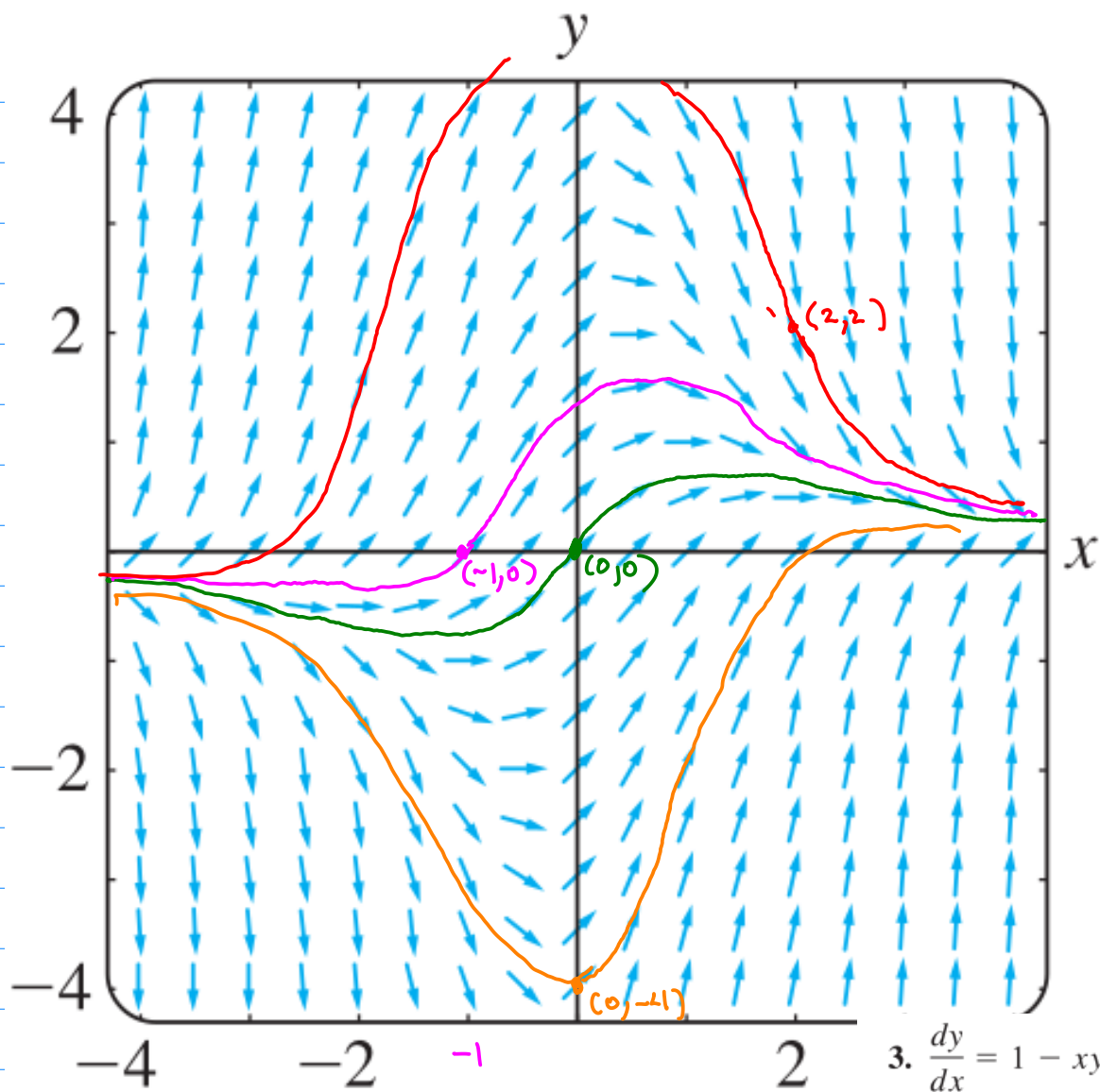




$$y' = \frac{xy}{5}$$

$$y(0) = 1$$

solution
that satisfies
the condition.



3. $\frac{dy}{dx} = 1 - xy$

• (a) $y(0) = 0$

• (b) $y(-1) = 0$

• (c) $y(2) = 2$

• (d) $y(0) = -4$

1. $(1-x)y'' - 4xy' + 5y = \cos x$

lin.

2

2. $x \frac{d^3y}{dx^3} - \left(\frac{dy}{dx}\right)^4 + y = 0$

nonlin.

3

3. $t^5y^{(4)} - t^3y'' + 6y = 0$

lin.

4

4. $\frac{d^2u}{dr^2} + \frac{du}{dr} + u = \cos(r+u)$

nonlin.

2

5. $\frac{d^2y}{dx^2} = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}$

nonlin

2

6. $\frac{d^2R}{dt^2} = -\frac{k}{R^2}$

nonlin

2

7. $(\sin \theta)y''' - (\cos \theta)y' = 2$

lin.

3

8. $\ddot{x} - \left(1 - \frac{\dot{x}^2}{3}\right)\dot{x} + x = 0$

nonlin.

2

means $\frac{d^4y}{dt^4}$

order

Show

$$-y^2 + \frac{1}{3}y^3 = \frac{1}{2}x^2 - \frac{1}{3}x^3 + C$$

is a solution to the differential equation

$$\frac{dy}{dx} = \frac{x(1-x)}{y(-2+y)}$$

Hint: use implicit differentiation

Autonomous differential equation is

$$y' = f(y)$$

\Rightarrow f does not depend on x .

Then the direction field does not depend on x .

Only need to look at one slice of the direction field to see what's happening...

$$26. \frac{dy}{dx} = y(2 - y)(4 - y)$$

