Pg 72 Chapter 2.5

Solve $(x^2 + y^2)\,dx + (x^2 - xy)\,dy = 0.$

$\boxed{y = ux}$

$dy = u\,dx + x\,du$

$(x^2 + y^2)\,dx + (x^2 - xy)\,(u\,dx + x\,du) = 0$

$(x^2 + (ux)^2)\,dx + x^2u\,dx - x(ux)u\,dx + x^3du - x^2(ux)\,du = 0$

$(x^2 + \cancel{x^2u^2} + x^2u - \cancel{x^2u^2})\,dx + (x^3 - x^3u)\,du = 0$

$(1+u)x^2\,dx + x^3(1-u)\,du = 0$

$\dfrac{x^2}{x^3}\,dx + \dfrac{(1-u)}{(1+u)}\,du = 0$

$\dfrac{1}{x}\,dx = \dfrac{-1+u}{1+u}\,du = \dfrac{-2+1+u}{1+u}\,du = \left(\dfrac{-2}{1+u} + 1\right)du$

$\int \dfrac{1}{x}\,dx = \int \left(\dfrac{-2}{1+u} + 1\right)du$

$\ln|x| = -2\ln|1+u| + u + C$

$y = ux$ in term of original variables $u = \dfrac{y}{x}$

implicit solution (good when $x \neq 0$ and $1 + \dfrac{y}{x} \neq 0$

$\ln|x| = -2\ln\left|1+\dfrac{y}{x}\right| + \dfrac{y}{x} + C$

Use properties of logarithms.

$$\ln|x| = -2\ln\left|1+\frac{y}{x}\right| + \frac{y}{x} + C$$

$$\ln|x| = -\ln\left|1+\frac{y}{x}\right|^2 + \frac{y}{x} + C$$

$$\underline{\ln|x| + \ln\left|1+\frac{y}{x}\right|^2} = \frac{y}{x} + C$$

$$\ln\left|x\left(1+\frac{y}{x}\right)^2\right| = \frac{y}{x} + C$$

$$\ln\left|\frac{x^2}{x}\left(1+\frac{y}{x}\right)^2\right| = \frac{y}{x} + C$$

$$\ln\left|\frac{(x+y)^2}{x}\right| = \frac{y}{x} + C$$

$$e^{\ln\left|\frac{(x+y)^2}{x}\right|} = e^{\left(\frac{y}{x} + C\right)}$$

$$\left|\frac{(x+y)^2}{x}\right| = e^{\frac{y}{x}} e^{C}$$

$$\frac{(x+y)^2}{x} = \pm e^{\frac{y}{x}} e^{C}$$ different constant $C_2 = \pm e^C$

$$\boxed{(x+y)^2 = C_2 x\, e^{\frac{y}{x}}}$$

advantage, no logs, or absolute values.

Next Chapter 2.6

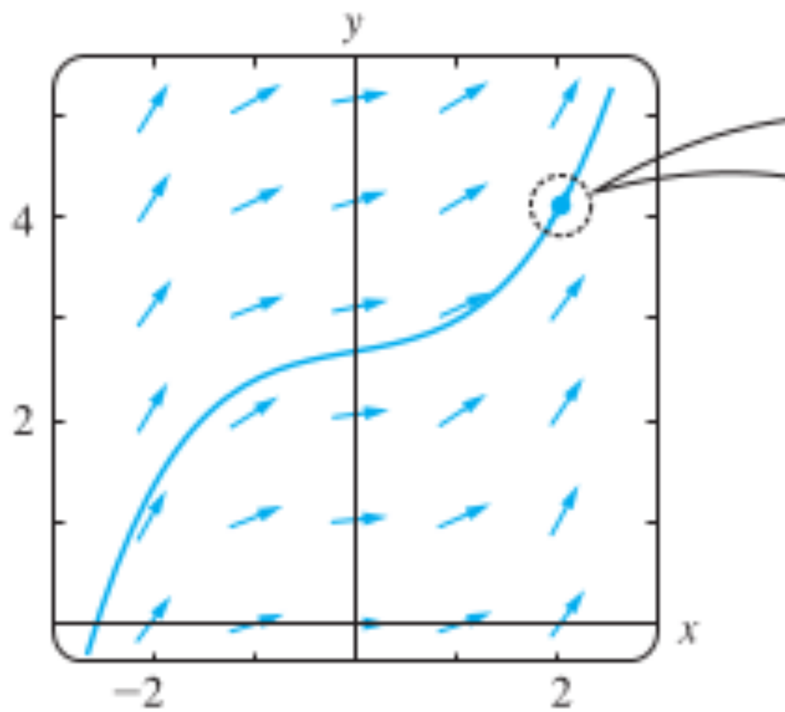
## 2.6 A NUMERICAL METHOD

differential equation

$$y' = f(x,y)$$

alg. condition (where to start)

$$y(x_0) = y_0$$




$$y' = f(x,y)$$

integrate the equation over the interval $[x_0, x_1]$

$$\int_{x_0}^{x_1} \frac{dy}{dx}\, dx = \int_{x_0}^{x_1} f(x,y)\, dx$$

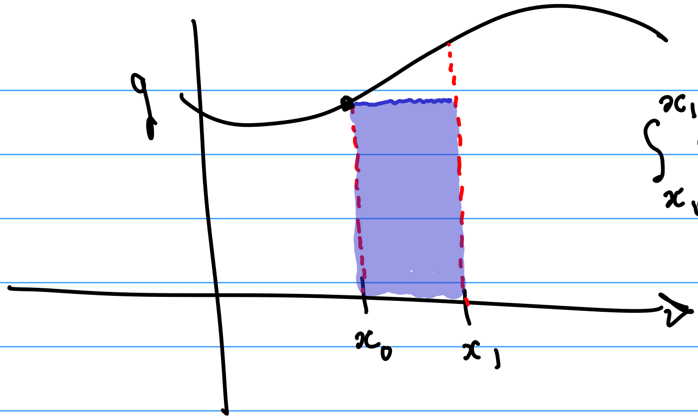
condition

$$y(x_0) = y_0$$

$$y(x_1) - y(x_0) = \int_{x_0}^{x_1} f(x, y(x))\, dx$$

$$y(x_1) = y_0 + \int_{x_0}^{x_1} f(x, y(x))\, dx$$

$$f(x, y(x)) = g(x)$$




$$\int_{x_0}^{x_1} g(x)\,dx \approx g(x_0)(x_1 - x_0)$$

$$\approx f(x_0, y(x_0))(x_1 - x_0)$$

know this

$$\approx f(x_0, y_0)(x_1 - x_0)$$

$$y(x_1) = y_0 + \int_{x_0}^{x_1} f(x, y(x))\,dx \approx y_0 + f(x_0, y_0)(x_1 - x_0)$$

Idea.. let $y_1 \approx y(x_1)$ given above. Then

$$y_1 = y_0 + f(x_0, y_0)(x_1 - x_0)$$

Now do it again, integrate over interval $[x_1, x_2]$

$$y(x_2) - y(x_1) = \int_{x_1}^{x_2} f(x, y(x))\,dx \approx f(x_1, y(x_1))(x_2 - x_1)$$

$$\approx \boxed{f(x_1, y_1)(x_2 - x_1)}$$

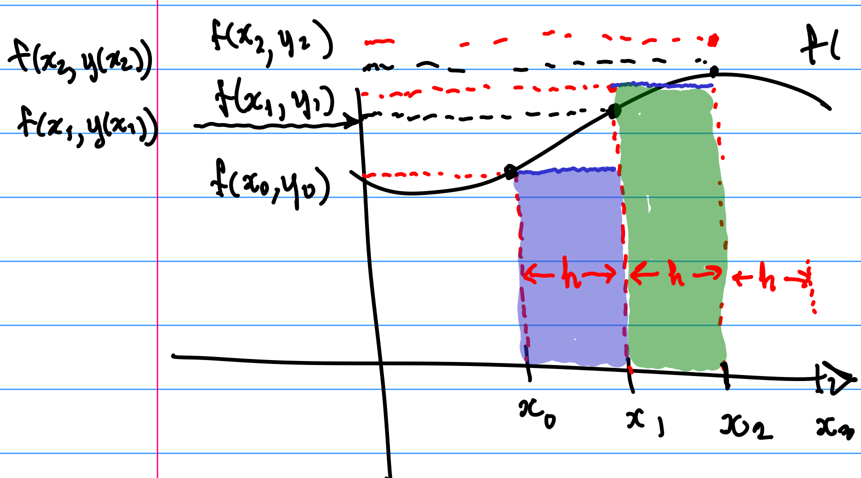
Therefore let

$$y_2 = y_1 + f(x_1, y_1)(x_2 - x_1)$$

and this gives the approximation $y_2 \approx y(x_2)$

$y_1 \approx y(x_1)$

$y_2 \approx y(x_2)$




define a grid on the $x$-axis

$$x_n = x_0 + hn$$

equally spaced.

$x_n = x_0 + hn$

$y_0 = y(x_0)$ the condition given in the problem.

$y_1 = y_0 + f(x_0, y_0)(x_1 - x_0) = y_0 + hf(x_0, y_0)$

$y_2 = y_1 + h f(x_1, y_1)$

$y_3 = y_2 + h f(x_2, y_2)$

$\vdots$

$y_{n+1} = y_n + h f(x_n, y_n)$

$$y' = 0.1\sqrt{y} + 0.4x^2, \quad y(2) = 4.$$

$f(x,y) = 0.1\sqrt{y} + 0.4x^2$ $\qquad x_0 = 2 \qquad y_0 = 4$

$x_1 = 2.1$

Try $h = 0.1$ and then compute

$y(x_1) \approx y_1 = y_0 + hf(x_0, y_0)$

$y(2.1) \approx y_1 = 4 + 0.1\left(0.1\sqrt{4} + 0.4(2)^2\right) =$ compute this next time...