Higher order $\boxed{\text{linear}}$ ordinary differential equations

2nd order...

linear in y...

$$y'' + 4y' + 7y = 0$$

for the Idea to work need to have 0 here...

(If not 0 we'll use linearity later)

Idea: Guess and check an exponential function:

$y = e^{rx}$ plug it in... $y' = re^{rx}$ $y'' = r^2 e^{rx}$

$$r^2 e^{rx} + 4re^{rx} + 7e^{rx} = 0\,e^{rx}$$

$$r^2 + 4r + 7 = 0$$

$a=1$ $b=4$ $c=7$

$$\begin{array}{r} 28 \\ 16 \\ \hline 12 \end{array}$$

Solve quadratic equation for r.

$$r = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-4 \pm \sqrt{16-28}}{2} = -2 \pm \frac{\sqrt{-12}}{2}$$

$$= -2 \pm \sqrt{-3} = -2 \pm i\sqrt{3}$$

Solutions to the differential equation are

$y_1 = e^{(-2+i\sqrt{3})x}$ $y_2 = e^{(-2-i\sqrt{3})x}$

one solution for each root of the quadratic equation...

what is exponential of an imaginary number?

or even a matrix?

what is an exponential function anyway?

① e^x is limit of compound interest (continuously compounded).

② Solution to the ODE $y' = y$, $y(0) = 1$

③ Inverse of $\ln x$.

what is $\ln x$?, an area given by an integral

$$\ln x = \int_1^x \frac{1}{t}\,dt.$$

④ Power series

$$e^x = 1 + x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \frac{1}{4!}x^4 + \cdots$$

$2! = 1 \cdot 2$

$3! = 1 \cdot 2 \cdot 3$

$4! = 1 \cdot 2 \cdot 3 \cdot 4$