Coming up:

Mar 4 Quiz 5 on $\underbrace{2.6,\ 9.1,\ 9.2}$

numerical techniques to approx. solution.

pratice for quiz... { Section 2.6#1,3,5,7
Section 9.1#1,4,9
Section 9.2#5,9,11

Mar 11 Midterm Comprehensive

(1) all quizzes

(2) Sections up to 4.4 on Syllabus (includes 9.1 & 9.2)

(3) Sample test over the weekend

22. $y''' - 6y'' + 12y' - 8y = 0$

☑ Linear
☑ constant coefficients
☑ Homogeneous

Guess: $y = e^{rx}$

$y' = re^{rx}$ $\quad y'' = r^2 e^{rx}$ $\quad y''' = r^3 e^{rx}$

Characteristic equation for r:

$r^3 - 6r^2 + 12r - 8 = 0$

$\pm 1, \pm 2, \pm 4, \pm 8$

$2^3 - 6\cdot 2^2 + 12\cdot 2 - 8 = 8 - 24 + 24 - 8$

$(r-2)(r^2 - 4r + 4) = 0$

$(r-2)(r-2)(r-2) = 0 \qquad r = 2$ mult 3

Solution $y_1 = e^{2x}$ but what is y_2 and y_3?

$$\begin{array}{r} r^2 - 4r + 4 \\ r-2 \overline{\big) r^3 - 6r^2 + 12r - 8} \\ \underline{r^3 - 2r^2} \\ -4r^2 + 12r - 8 \\ \underline{-4r^2 + 8r} \\ 4r - 8 \\ \underline{4r - 8} \\ 0 \end{array}$$

already knew remainder was zero

Last time we set $y_2 = u e^{2x}$ and solved for u.

22. $y''' - 6y'' + 12y' - 8y = 0$

$y = u e^{2x}$ $\quad y' = u' e^{2x} + 2u e^{2x}$

$y'' = u'' e^{2x} + \underbrace{2u' e^{2x} + 2u' e^{2x}} + 4u e^{2x}$

$= u'' e^{2x} + 4u' e^{2x} + 4u e^{2x}$

$y''' = u''' e^{2x} + 2u'' e^{2x} + 4u'' e^{2x} + 8u' e^{2x} + 4u' e^{2x} + 8u e^{2x}$

skip steps

$u''' = 0$ so $u = c_1 + c_2 x + c_3 x^2$

Thus $y = (c_1 + c_2 x + c_3 x^2) e^{2x} = c_1 y_1 + c_2 y_2 + c_3 y_3$

where $y_1 = e^{2x}$, $y_2 = x e^{2x}$, $y_3 = x^2 e^{2x}$

Try plugging in x for u:

Could also check this solution to see if it works...

22. $y''' - 6y'' + 12y' - 8y = 0$

$y = xe^{2x} \qquad y' = 1e^{2x} + 2xe^{2x}$

$y'' = 2e^{2x} + 2e^{2x} + 4xe^{2x}$

$= 4e^{2x} + 4xe^{2x}$

$y''' = 8e^{2x} + 4e^{2x} + 8xe^{2x}$

$y''' = 12e^{2x} + 8xe^{2x}$

$y''' = 12e^{2x} + 8xe^{2x}$

$-6y'' = -24e^{2x} - 24xe^{2x}$

$12y' = 12e^{2x} + 24xe^{2x}$

$-8y = -8xe^{2x}$

$y''' - 6y'' + 12y' - 8y = 0$

General solution $y = c_1y_1 + c_2y_2 + c_3y_3$

where $y_1 = e^{2x}$, $y_2 = xe^{2x}$, $y_3 = x^2e^{2x}$

31. $\dfrac{d^2y}{dt^2} - 4\dfrac{dy}{dt} - 5y = 0, \quad y(1) = 0, y'(1) = 2$

$$r^2 - 4r - 5 = 0$$

$$(r-5)(r+1) = 0$$

$$r = 1, 5$$

$$\boxed{y(x) = c_1 e^{x} + c_2 e^{5x}}$$ general solution

Now find unique solution

$y(1) = 0, y'(1) = 2$

$$y'(x) = c_1 e^{x} + 5c_2 e^{5x}$$

$$y(1) = c_1 e^1 + c_2 e^5 = 0$$

$$y'(1) = c_1 e^1 + 5c_2 e^5 = 2$$

Two equations in 2 unknowns

$$c_1 e^1 + c_2 e^5 = 0 \qquad c_1 = -c_2 e^4$$

$$c_1 e^1 + 5c_2 e^5 = 2$$

$$-c_2 e^5 + 5c_2 e^5 = 2$$

$$c_2(5e^5 - e^5) = 2 \qquad c_2 = \frac{2}{5e^5 - e^5} = \frac{2}{4e^5}$$

$$c_1 = -\frac{2}{4e^5} e^4 = -\frac{2}{4e}$$

Unique solution

$$y = -\frac{1}{2e} e^{x} + \frac{1}{2e^5} e^{5x}$$

TABLE 4.4.1 Trial Particular Solutions

$g(x)$	Form of y_p
1. 1 (any constant)	A
2. $5x + 7$	$Ax + B$
3. $3x^2 - 2$	$Ax^2 + Bx + C$
4. $x^3 - x + 1$	$Ax^3 + Bx^2 + Cx + E$
5. $\sin 4x$	$A\cos 4x + B\sin 4x$
6. $\cos 4x$	$A\cos 4x + B\sin 4x$
7. e^{5x}	Ae^{5x}
8. $(9x - 2)e^{5x}$	$(Ax + B)e^{5x}$
9. x^2e^{5x}	$(Ax^2 + Bx + C)e^{5x}$
10. $e^{3x}\sin 4x$	$Ae^{3x}\cos 4x + Be^{3x}\sin 4x$
11. $5x^2\sin 4x$	$(Ax^2 + Bx + C)\cos 4x + (Ex^2 + Fx + G)\sin 4x$
12. $xe^{3x}\cos 4x$	$(Ax + B)e^{3x}\cos 4x + (Cx + E)e^{3x}\sin 4x$

Now consider inhomogeneous

6. $y'' - 8y' + 20y = \overbrace{\underbrace{100x^2}_{g_1} - \underbrace{26xe^x}_{g_2}}^{g}$

☑ Linear
☑ constant coefficients
☑ Homogeneous

$y_h'' - 8y_h' + 20y_h = 0$

$y_{P_1}'' - 8y_{P_1}' + 20\, y_{P_1} = 100x^2$

$y_{P_2}'' - 8y_{P_2}' + 20\, y_{P_2} = -26x\, e^{x}$

$(y_{P_1} + y_{P_2})'' - 8(y_{P_1} + y_{P_2})' + 2\,(y_{P_1} + y_2) = 100x^2 - 26x\, e^{x} + 0$

solution to original $y = y_{P_1} + y_{P_2} + y_h$