$$f(t) = 3t^2 - e^{-t} - \int_0^t f(\tau) e^{t-\tau} d\tau$$

$$f(t) = \mathcal{L}^{-1}\left\{ \frac{6}{s^3} - \frac{2}{s+1} - \frac{6}{s^4} + \frac{1}{s} \right\}$$

$$= 3\mathcal{L}^{-1}\left\{ \frac{2!}{s^{2+1}} \right\} - 2\mathcal{L}^{-1}\left\{ \frac{1}{s-(-1)} \right\} - \mathcal{L}^{-1}\left\{ \frac{3!}{s^{3+1}} \right\} + \mathcal{L}^{-1}\left\{ \frac{1}{s} \right\}$$

$$f(t) = 3t^2 - 2e^{-t} - t^3 + 1$$

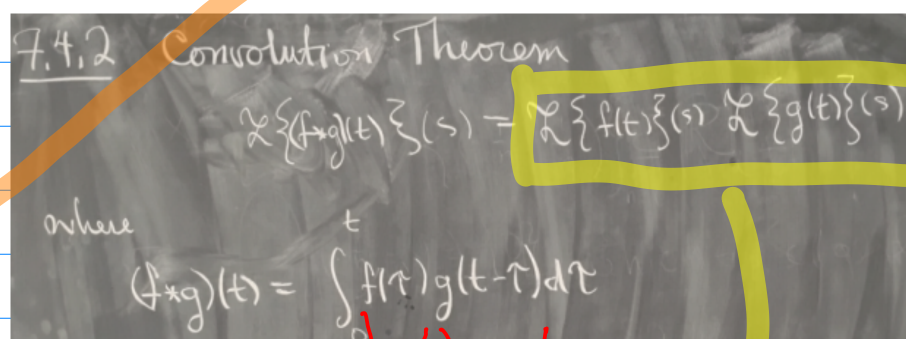
$$\boxed{L \frac{di}{dt} + R i(t) + \frac{1}{C} \int_0^t i(\tau)\, d\tau = E(t).}$$

$$E(t) = 120t - 120t\, \mathcal{U}(t-1).$$

$L = 0.1, \quad R = 2, \quad C = 0.1, \quad i(0) = 0$

Take Laplace transforms of both sides

$$L\, \mathcal{L}\left\{ \frac{di}{dt} \right\}(s) + R \underbrace{\mathcal{L}\{i(t)\}(s)}_{I(s)} + \frac{1}{C} \mathcal{L}\left\{ \int_0^t i(\tau)\, d\tau \right\}(s) = \mathcal{L}\{E(t)\}(s)$$



$i(\tau)$ constant function $g(t) = 1$

$$\mathcal{L}\left\{ \underbrace{\int_0^t i(\tau)\, d\tau}_{\text{convolution}} \right\}(s)$$

$i * 1$

$$\mathcal{L}\left\{ \frac{di}{dt} \right\}(s) = -i(0) + s\mathcal{L}\{i(t)\}(s) = -i(0) + sI(s)$$

$$-L\,i(0) + sL\,I(s) + R\,I(s) + \frac{1}{C}\mathcal{L}\{i * 1\}(s) = \mathcal{L}\{E(t)\}(s)$$

$$-L\,i(0) + sL\,I(s) + R\,I(s) + \frac{1}{C}I(s)\,\mathcal{L}\{1\}(s) = \mathcal{L}\{E(t)\}(s)$$

Solve for $I(s)$

$$I(s)\left(sL + R + \frac{1}{C}\mathcal{L}\{1\}(s)\right) = \mathcal{L}\{E(t)\}(s) + L\,i(0)$$

$$I(s) = \frac{\mathcal{L}\{E(t)\}(s) + L\,i(0)}{sL + R + \frac{1}{C}\mathcal{L}\{1\}(s)}$$

Note we are solving an equation with both derivatives and integrals

$$L\frac{di}{dt} + Ri(t) + \frac{1}{C}\int_0^t i(\tau)\,d\tau = E(t)$$

If I differentiate the equation it becomes

$$\frac{d}{dt}\left(L\frac{di}{dt} + Ri(t) + \frac{1}{C}\int_0^t i(\tau)\,d\tau\right) = \frac{d}{dt}\left(E(t)\right)$$

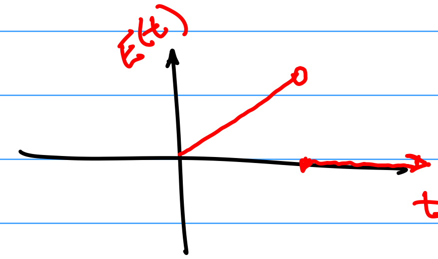
$$L\frac{d^2i}{dt^2} + R\frac{di}{dt} + \frac{1}{C}i = E'(t)$$ ← This gives a 2nd order linear differential equation as long as E(t) was differentiable ... but

$$E(t) = 120t - 120t\,\mathcal{U}(t-1). = 120t\left(1 - \mathcal{U}(t-1)\right)$$

$$\mathcal{U}(t-1) = \begin{cases} 0 & \text{for } t < 1 \\ 1 & \text{for } t \geq 1 \end{cases}$$

$$1-\mathcal{U}(t-1) = \begin{cases} 1 & \text{for } t<1 \\ 0 & \text{for } t\geq 1 \end{cases}$$

$$E(t) = \begin{cases} 120t & \text{for } t<1 \\ 0 & \text{for } t\geq 1 \end{cases}$$



That's why we used Laplace transforms...

$$I(s) = \frac{\mathcal{L}\{E(t)\}(s) + L\,i(0)}{sL + R + \frac{1}{C}\mathcal{L}\{1\}(s)}$$

$a=1$

$$\mathcal{L}\{g(t)\mathcal{U}(t-a)\} = e^{-as}\mathcal{L}\{g(t+a)\}.$$

$$\mathcal{L}\{E(t)\}(s) = \mathcal{L}\{120t - 120t\,\mathcal{U}(t-1)\}(s)$$

$$= 120\,\mathcal{L}\{t\}(s) - 120\,\mathcal{L}\{t\,\mathcal{U}(t-1)\}(s)$$

$$= 120\frac{1}{s^2} - 120\,e^{-s}\mathcal{L}\{t+1\}(s).$$

$$= 120\frac{1}{s^2} - 120\left\{\frac{e^{-s}}{s^2} + \frac{e^{-s}}{s}\right\}$$

$L=0.1$, $R=2$, $C=0.1$, $i(0)=0$

$$I(s) = \frac{\mathcal{L}\{E(t)\}(s) + L\,i(0)}{sL + R + \frac{1}{C}\mathcal{L}\{1\}(s)}$$

$$= \frac{120\frac{1}{s^2} - 120\left\{\frac{e^{-s}}{s^2} + \frac{e^{-s}}{s}\right\}}{0.1s + 2 + 10\frac{1}{s}}$$

$$0.1s + 2 + 10\frac{1}{s} = \frac{0.1s^2 + 2s + 10}{s}$$

$$I(s) = \frac{s}{0.1s^2 + 2s + 10}\left(120\frac{1}{s^2} - 120\left\{\frac{e^{-s}}{s^2} + \frac{e^{-s}}{s}\right\}\right)$$

$$= \frac{120\left(\frac{1}{s} - \frac{e^{-s}}{s} - e^{-s}\right)}{0.1s^2 + 2s + 10} \cdot \frac{10}{10} = \frac{1200\left(\frac{1}{s} - \frac{e^{-s}}{s} - e^{-s}\right)}{\underbrace{s^2 + 20s + 100}_{(s+10)^2}}$$

$$= 1200\left(\frac{1}{s(s+10)^2} - \frac{e^{-s}}{s(s+10)^2} - \frac{e^{s}}{(s+10)^2}\right)$$

By partial fractions,

$$I(s) = 1200\left[\frac{1/100}{s} - \frac{1/100}{s+10} - \frac{1/10}{(s+10)^2} - \frac{1/100}{s}e^{-s} + \frac{1/100}{s+10}e^{-s} + \frac{1/10}{(s+10)^2}e^{-s} - \frac{1}{(s+10)^2}e^{-s}\right].$$

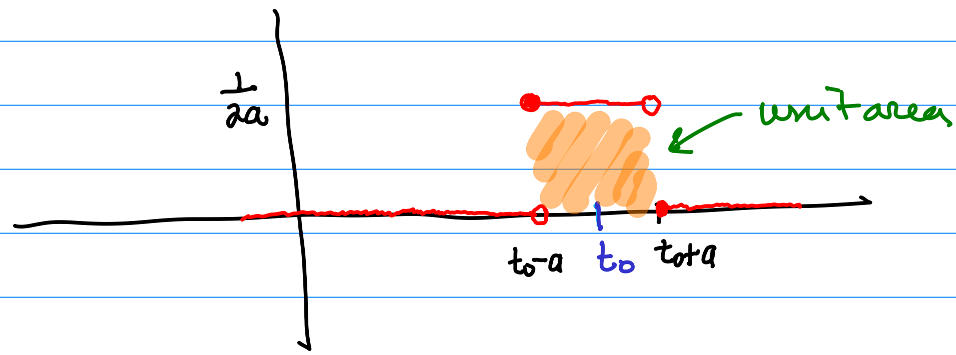
From the inverse form of the second translation theorem, (15) of Section 7.3, we finally obtain

$$i(t) = 12[1 - \mathscr{U}(t-1)] - 12[e^{-10t} - e^{-10(t-1)}\mathscr{U}(t-1)] - 120te^{-10t} - 1080(t-1)e^{-10(t-1)}\mathscr{U}(t-1).$$

Check this from page 306 in the text... active reading by filling in the missing steps from what's written.

7.5 THE DIRAC DELTA FUNCTION

$$\delta_a(t-t_0) = \begin{cases} 0 & \text{for} \quad t < t_0 - a \\ \frac{1}{2a} & \text{for} \quad t_0 - a \leq t < t_0 + a \\ 0 & \text{fo} \quad t \geq t_0 - a \end{cases}$$



$t_0 > 0$

$$\int_0^\infty \delta_a(t-t_0)\, dx = 2a \frac{1}{2a} = 1$$