#1. $\frac{dy}{dx} = e^{5x}e^{3y}$; $\int e^{-3y}\,dy = \int e^{5x}\,dx$

$\frac{1}{-3}e^{-3y} = \frac{1}{5}e^{5x} + C$; $e^{-3y} = \frac{-3}{5}e^{5x} + c$

$-3y = \ln\left(\frac{-3}{5}e^{5x} + c\right)$; $y = -\frac{1}{3}\ln\left(c - \frac{3}{5}e^{5x}\right)$

#2 $y' + y = e^{3x}$; $(ye^{x})' = e^{4x}$; $ye^{x} = \frac{1}{4}e^{4x} + C$

$y = \frac{1}{4}e^{3x} + Ce^{-x}$; $5 = y(0) = \frac{1}{4} + C$; $C = 5 - \frac{1}{4} = \frac{19}{4}$

$y = \frac{1}{4}e^{3x} + \frac{19}{4}e^{-x}$.

#3 $(2xy^2 - 5)\,dx + (2x^2y + 4)\,dy = 0$; $M = 2xy^2 - 5$; $N = 2x^2y + 4$

$\frac{\partial M}{\partial y} = 4xy$; $\frac{\partial N}{\partial x} = 4xy$ ← same so exact.

$h(x,y) = \int (2xy^2 - 5)\,dx = x^2y^2 - 5x + g(y)$

$\frac{\partial h}{\partial y} = 2x^2y + g'(y) = 2x^2y + 4$; $g'(y) = 4$; $g(y) = 4y + c$.

$h(x,y) = x^2y^2 - 5x + 4y + c$; soln: $x^2y^2 - 5x + 4y = C$.

#4 $4xy\,dx + (4y+6x^2)dy = 0$

$$\frac{\partial}{\partial y}(\mu 4xy) = \mu_y 4xy + \mu 4x$$

$$\frac{\partial}{\partial x}\left(\mu(4y+6x^2)\right) = \underbrace{\mu_x(4y+6x^2)}_{=0} + \mu\, 12x$$

Assume $\mu_x = 0$ so that $\mu = \mu(y)$ only. Then

$$\frac{d\mu}{dy} 4\cancel{x}y + \mu 4\cancel{x} = \mu\, 12\cancel{x}$$

$$\frac{d\mu}{dy} \cancel{4}y = \overset{2}{\cancel{8}}\mu\,; \quad \int \frac{d\mu}{\mu} = \int \frac{2\,dy}{y}$$

$$\ln|\mu| = 2\ln|y| + C; \quad \ln|\mu| = \ln y^2 + C$$

$$|\mu| = y^2 e^C\,; \quad \mu = Cy^2$$

Therefore multiply the original equation by y^2

$$4xy^3\,dx + (4y^3 + 6x^2y^2)dy = 0$$

$$h(x,y) = \int 4xy^3\,dx = 2x^2y^3 + g(y)$$

$$\frac{\partial h}{\partial y} = \cancel{6x^2y^2} + g'(y) = 4y^3 + \cancel{6x^2y^2}$$

$$g'(y) = 4y^3; \quad g(y) = y^4 + C$$

$h(x,y) = 2x^2y^3 + y^4 + C$; Soln: $2x^2y^3 + y^4 = C$.

#5 $(y^2+yx)dx - x^2 dy = 0 \qquad y' = \frac{y^2+yx}{x^2}$

Let $y = ux$; $\quad y' = u'x + u$

$\frac{y^2+yx}{x^2} = u'x+u$; $\quad \left(\frac{y}{x}\right)^2 + \left(\frac{y}{x}\right) = u'x+u$

$u^2 + \cancel{u} = u'x + \cancel{u}$; $\quad \frac{du}{dx}x = u^2$

$\int \frac{du}{u^2} = \int \frac{dx}{x}$; $\quad -u^{-1} = \ln|x| + C$

$-\frac{x}{y} = \ln|x| + C$; $\quad y = \frac{-x}{\ln|x|+C}$

#6 $x^2y' - 2xy = 5y^4$; $\quad y' - \frac{2}{x}y = \frac{5}{x^2}y^4$; $\quad n=4$

$u = y^{1-n} = y^{-3}$; $\quad u' = -3y^{-4}y'$

$u' = -3y^{-4}\left(\frac{2}{x}y + \frac{5}{x^2}y^4\right) = -\frac{6}{x}y^{-3} - \frac{15}{x^2}$

$u' + \frac{6}{x}u = -\frac{15}{x^2}$; $\quad$ mult by $e^{\int \frac{6}{x}dx} = x^6$

$x^6u' + 6x^5u = -15x^4$; $\quad (x^6u)' = -15x^4$

$x^6u = -3x^5 + C$; $\quad x^6y^{-3} = -3x^5 + C$

$\frac{1}{y^3} = \frac{-3}{x} + \frac{C}{x^6}$; Solve for C using $y(1) = \frac{1}{3}$

$27 = -3 + C$; $\quad C = 30.$

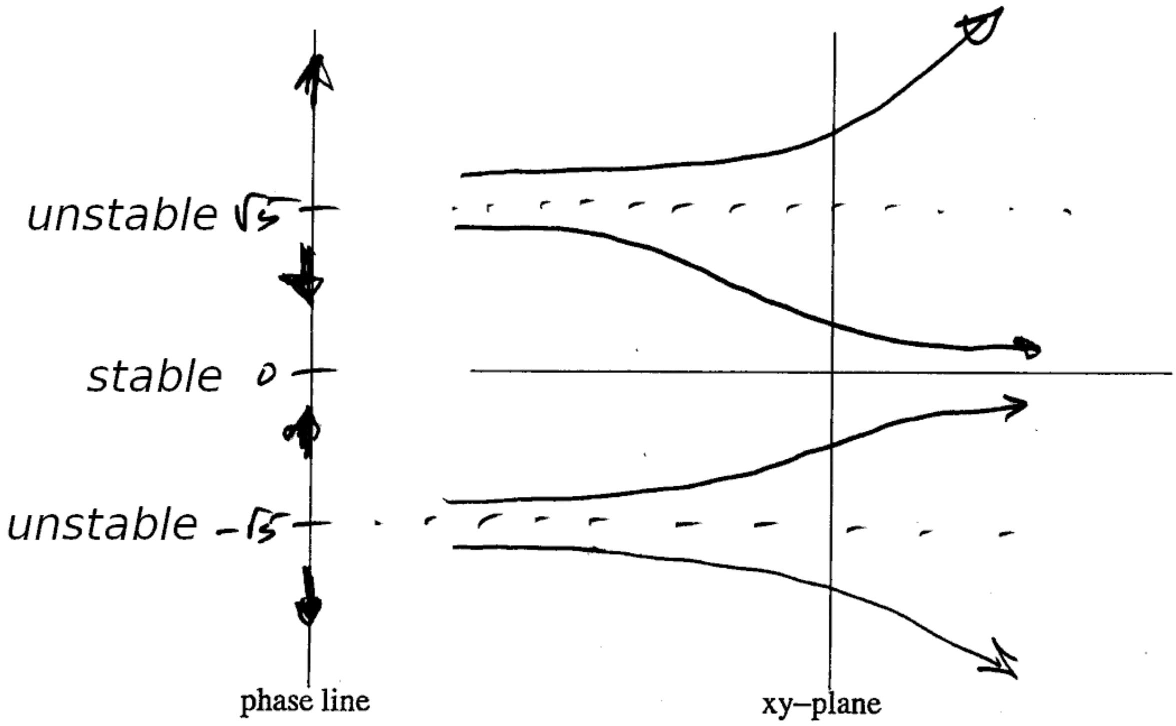
$\frac{1}{y^3} = \frac{-3}{x} + \frac{30}{x^6}$; $\quad y^3 = \frac{x^6}{30-3x^5}$; $\quad y = \frac{x^2}{\sqrt[3]{30-3x^5}}$

$y = x^2(30-3x^5)^{-1/3}$.

Quiz 1

Math 285 ~~Sample Exam~~ 2 Version B

7. Draw a phase portrait and solution curves for the autonomous first-order ordinary differential equation $y' = y^3 - 5y$ below. Label the stationary points and determine whether they are stable, unstable or semi-stable.



$$y^3 - 5y = y(y^2-5) = y(y-\sqrt{5})(y+\sqrt{5})$$

$$y = 0, -\sqrt{5}, \sqrt{5}$$