1. Use the convolution theorem to evaluate the Laplace transform

\[ F(s) = \mathcal{L}\left\{ \int_0^t e^{t-\tau} d\tau \right\}(s). \]

The Laplace transform is

\[ F(s) = \]

2. Suppose

\[ A = \begin{bmatrix} 1 & 6 \\ 7 & 11 \\ 10 & 12 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} -6 & 8 & -3 \\ 1 & -3 & 2 \end{bmatrix} \]

Find the products

\[ AB = \]
\[ BA = \]

3. Consider the matrix

\[ A = \begin{bmatrix} -1 & 1 & 0 \\ 1 & 3 & 1 \\ 0 & 4 & -1 \end{bmatrix} \]

with eigenvalues

\[ \lambda_1 = 4, \quad \lambda_2 = -2 \quad \text{and} \quad \lambda_3 = -1 \]

and corresponding eigenvectors

\[ K_1 = \begin{bmatrix} 1 \\ 5 \\ 4 \end{bmatrix}, \quad K_2 = \begin{bmatrix} 1 \\ -1 \\ 4 \end{bmatrix} \quad \text{and} \quad K_3 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}. \]

Find the general solution of the matrix differential equation \( X' = AX \).

\[ X(t) = \]
4. Consider the matrix 

\[ A = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix} \]

with eigenvalues 
\[ \lambda_1 = 1 + 2i \quad \text{and} \quad \lambda_2 = 1 - 2i \]

and corresponding eigenvectors 
\[ K_1 = \begin{bmatrix} 1 \\ i \end{bmatrix}, \quad \text{and} \quad K_2 = \begin{bmatrix} i \\ 1 \end{bmatrix}. \]

Find the unique solution of the matrix differential equation 

\[ X' = AX, \quad X(0) = \begin{bmatrix} 3 \\ 4 \end{bmatrix}. \]

\[ X(t) = \]

---

5. Consider the matrix 

\[ A = \begin{bmatrix} 5 & -1 \\ 4 & 1 \end{bmatrix} \]

with an eigenvalue 
\[ \lambda_1 = 3 \quad \text{of multiplicity} \quad 2 \]

and corresponding eigenvector 
\[ K_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}. \]

Note that this is the case where there is no second eigenvector. Find the general solution of the matrix differential equation \( X' = AX \).

\[ X(t) = \]