Math 285: Sample Midterm Version A

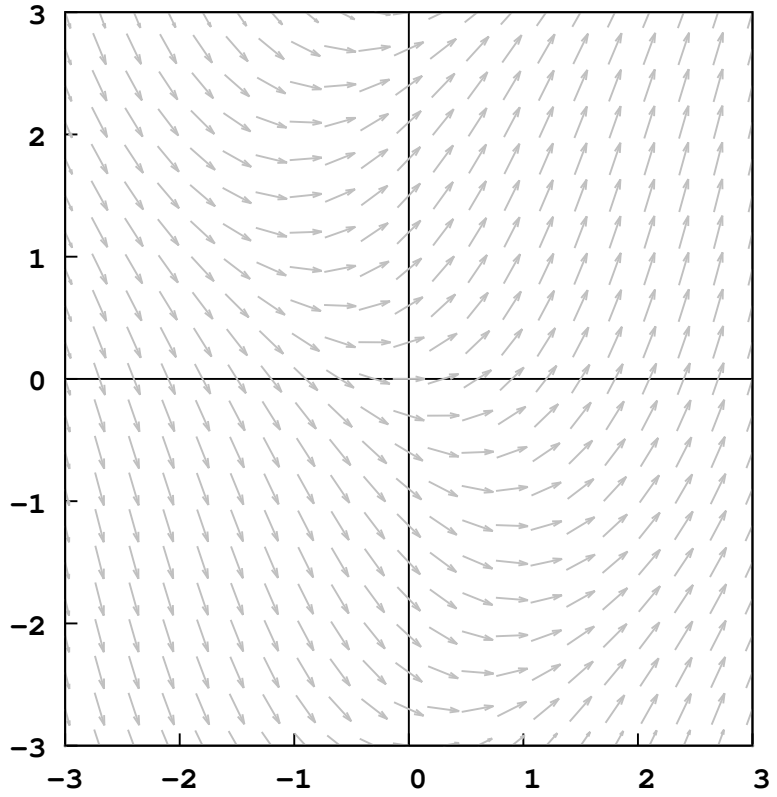
**1.** State the order of the given ordinary differential equations and whether the equations are linear or nonlinear.

**(i)** $\dfrac{dy}{dx} = x^2 + y^2$

**(ii)** $y''' - 6y'' + 11y' - 6y = \sin x$

**2.** Check whether $y = e^x \sin x$ is a solution to $y'' - 2y' + 3y = 0$. Show your work explaining why or why not.

**3.** The direction field for the differential equation $dy/dx = x + \sin y$ is given. Sketch a solution curve passing through the point $y(-1) = 0$.




**4.** Use separation of variables to find an implicit solution to

$$\frac{dy}{dx} = \frac{x}{1+y^2}$$

**5.** Find the unique solution to the linear differential equation

$$\frac{dy}{dx} + y = e^{3x} \qquad \text{such that} \qquad y(0) = 1.$$

**6.** Determine whether the functions

$$f_1(x) = 1 + x, \qquad f_2(x) = x \qquad \text{and} \qquad f_3(x) = x^2$$

are linearly independent on the interval $(-\infty, \infty)$.

**7.** Substitute $y = xu$ to reduce the homogeneous equation

$$(y^2 + yx)dx + x^2 dy = 0$$

to a separable differential equation in $u$. Don't solve the equation in $u$.

**8.** Solve the differential equation

$$(2y^2 + 3x)\,dx + 2xy\,dy = 0$$

by finding an integrating factor $\mu$ that depends only on $x$ so it is exact.

**9.** Consider the differential equation

$$y' = 4x - 2y \qquad \text{with} \qquad y(0) = 2.$$

**(i)** Compute one step of Euler's method with $h = 0.1$ to obtain a four-decimal approximation of $y(0.1)$.

**(ii)** Compute one step of the improved Euler's method with $h = 0.1$ to obtain a four-decimal approximation of $y(0.1)$.

**(iii)** Compute one step of the fourth-order Runge–Kutta method with $h = 0.1$ to obtain a four-decimal approximation of $y(0.1)$.

**10.** The function $y_1(x) = x^4$ satisfies $x^2y'' - 7xy' + 16y = 0$ for $x > 0$. Use reduction of order to find a second solution $y_2(x)$.

**11.** Solve $y'' - 2y' + 5y = e^x$ by undetermined coefficients.

## Summary of Numerical Methods

Euler's Method:

$$y_{n+1} = y_n + hf(x_n, y_n).$$

Improved Euler's Method (RK2):

$$\begin{aligned} k_1 &= f(x_n, y_n) \\ k_2 &= f(x_n + h, y_n + hk_1) \\ y_{n+1} &= y_n + (h/2)(k_1 + k_2). \end{aligned}$$

Fourth-order Runge–Kutta Method (RK4):

$$\begin{aligned} k_1 &= f(x_n, y_n) \\ k_2 &= f(x_n + \tfrac{1}{2}h, y_n + \tfrac{1}{2}hk_1) \\ k_3 &= f(x_n + \tfrac{1}{2}h, y_n + \tfrac{1}{2}hk_2) \\ k_4 &= f(x_n + h, y_n + hk_3) \\ y_{n+1} &= y_n + \tfrac{1}{6}h(k_1 + 2k_2 + 2k_3 + k_4). \end{aligned}$$

**TABLE 4.4.1** Trial Particular Solutions

| $g(x)$ | Form of $y_p$ |
|---|---|
| **1.** 1 (any constant) | $A$ |
| **2.** $5x + 7$ | $Ax + B$ |
| **3.** $3x^2 - 2$ | $Ax^2 + Bx + C$ |
| **4.** $x^3 - x + 1$ | $Ax^3 + Bx^2 + Cx + E$ |
| **5.** $\sin 4x$ | $A\cos 4x + B\sin 4x$ |
| **6.** $\cos 4x$ | $A\cos 4x + B\sin 4x$ |
| **7.** $e^{5x}$ | $Ae^{5x}$ |
| **8.** $(9x - 2)e^{5x}$ | $(Ax + B)e^{5x}$ |
| **9.** $x^2e^{5x}$ | $(Ax^2 + Bx + C)e^{5x}$ |
| **10.** $e^{3x}\sin 4x$ | $Ae^{3x}\cos 4x + Be^{3x}\sin 4x$ |
| **11.** $5x^2\sin 4x$ | $(Ax^2 + Bx + C)\cos 4x + (Ex^2 + Fx + G)\sin 4x$ |
| **12.** $xe^{3x}\cos 4x$ | $(Ax + B)e^{3x}\cos 4x + (Cx + E)e^{3x}\sin 4x$ |